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APPLIED MECHANICS REVIEWS

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MARTIN GOLAND *Editor*

SEPTEMBER 1953

VORTEX STREETS IN INCOMPRESSIBLE MEDIA¹

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THIS review refers to vortex streets in liquids and in those regions of gaseous media where effects of compressibility may be ignored. Effects of viscosity are usually neglected. Papers are reviewed in chronological order.

STABILITY OF VORTEX STREET AND RESISTANCE

Mallock explained that aeolian tones investigated by Strouhal, Rayleigh, and others are due to vortices. Bénard mentioned that Reynolds and Brillouin had already noticed vortices behind bodies. Von Kármán and Rubach showed that a double row of vortices is stable for only one geometrical configuration. Föppl investigated vortex-pair placed symmetrically, and Rubach pointed out that there are two symmetrical vortices at the back of an obstacle at the beginning of the motion. Heisenberg obtained characteristic dimensions different from those given by von Kármán, but Prandtl criticized his assumptions. Rolling up of vortex sheet was calculated by Prandtl. Dryden and Heald found that turbulence in main stream affects vortex configuration, and Fage and Johansen that geometrical characteristics change downstream.

Rayleigh calculated the sides of cellular vortices formed when a layer of liquid is heated from below. His values agree well with those given by Bénard. Syngé investigated mathematically the thrust experienced by cylinder in a stream. Goldstein noticed that calculation of two-dimensional streaming past cylinder with vortices by Bickley is deprived of practical importance by the unstable character of the motion. Schlayer discussed the stability of vortex street in three dimensions. Walton experimented with rods in water, and Fage and Johansen with flat plate, airfoil, cylinder, and wedge in air stream. Glauert extended von Kármán's analysis to flow in a channel of finite breadth, and Pérès derived a formula analogous to that of Syngé. Rosenhead showed that the presence of channel walls causes geometrical changes and that the only arrangement of equally spaced double rows of vortices which is stable is von Kármán's street. Villat derived a formula for the resistance, using the theory of double row of vortices. This originated a discussion by Rosenhead-Villat. Rosenhead discussed the stability of von Kármán's street for vortices of finite sections and performed many tests behind cylinders in channels of different breadths. Levy and Hooker have shown that the vortex street is unstable if undisturbed velocity is not uniform, and Richards compared experiments of flow past an elliptic cylinder with mathematical theory. Hooker attributed

downstream variation in spacing ratio to viscosity. Schmieden showed that higher-order terms contribute to a weak instability of vortex street, and Dolapchiev presented analytical construction of stable and unstable paths described by a single point vortex. Kochin showed that von Kármán's condition of stability is necessary but not sufficient, and proposed a new definition of stability in the sense of Lyapunov. Maue derived the entire family of stable vortex systems, and the general problem of two-dimensional motion of vortices was attacked by Lin. Von Kármán determined an example of symmetrical unsteady flow behind a flat plate. Kurihara considered diffusion of turbulence energy in the wake, and Kaufmann the vortices having finite cross sections, Dolapchiev discussed the stability of an arbitrarily situated vortex street and conditions of stability in the sense of Kochin. Coddington, following suggestions of Wintner, considered the stability of infinite linear differential system, as applied to vortex streets. Some additional information may be found in the works of Villat, Lamb, and Goldstein.

FREQUENCY OF VORTEX FORMATION

This is usually expressed by means of Strouhal number S . Von Kármán measured S behind circular cylinder, and Krüger and Schmidtke used it in investigation of acoustic phenomena. Relf and Ower verified that frequency of the note heard is the frequency of eddies. Relf and Simmons did not detect periodic motion below Reynolds number equal to 100, but Richardson and Thom detected it down to $Re = 74$. Relf and Simmons, Dryden and Heald, and Camichel measured S behind cylinder. Bénard derived a formula for frequency of vortices having the same sense of rotation, and noticed a deviation between experiments and laws of dynamic similarity. Fage and Johansen investigated in detail the structure of a vortex sheet behind normal plate, airfoil, cylinder, and wedge in air stream. Bénard compared the results of his experiments of knife-blade shapes with those of Camichel, Dupin, and Teissié-Solier. He found that surface tension produces no marked influence on deviation from dynamic similarity. Tyler measured precisely the frequency behind various obstacles and suggested a formula for S which agrees well with that proposed by Rayleigh as a result of his analysis of observations of Strouhal. Winny investigated the wake of a sphere, and Blenk, Fuchs, and Liebers measured the frequency behind various bodies in water and air stream. Their results do not seem to agree with those cited by Abdrashitov. Ruedy considered oscillating lift, and Landweber experimented with vortex street behind a pair of cylinders. Krzywoblocki concluded that S is not constant in the wake behind various bodies, and Spivack investigated frequency behind two parallel cylinders.

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EDGE TONES

Presence of alternating vortices may be noted in study of edge tones produced when a plate is attacked by jet of air (Benton, Carrière, and remarks in Spivaak's paper).

SECONDARY VORTICES

These vortices, as distinguished from alternating vortices, are sometimes produced at the borders of wakes of sharp-edged plates (Courregelongue, Camichel and Escande, Prandtl, and remarks in Spivaak's paper).

RELATION OF VORTEX STREET TO TURBULENT WAKE

Certain remarks on that subject can be found in papers by Fage and Johansen (who noticed that change in spacing ratio is accompanied by diffusion of vorticity), by Dryden, Spivaak (vorticity appears downstream as isotropic turbulence), Hooker, Goldstein, Krzywoblocki, and others.

THREE-DIMENSIONAL VORTEX RINGS

The configuration of vortex rings behind three-dimensional bodies and their stability was investigated by Eden, Nisi and Porter, Lock, Levy and Forsdyke, Schmiedel, Jeffreys, Stanton and Marshall, Simmons and Dewey, Falkenhagen, Rosenhead (1932), Carter, Harrington, Möller, and others.

MODERN HYPOTHESES ON STRUCTURE OF WAKES

Townsend, using spectrum functions, suggested that a wake consists of a fully continuously turbulent core surrounded by a region of intermittent turbulence due to jets or billows of turbulent fluid from the core. Kovasznay concluded that a vortex street can be considered as an instability of laminar wake, and Batchelor thoroughly discussed Townsend's hypothesis. Roshko introduced the notion of subranges into which a wake may be divided: stable-, transition-, irregular-, and turbulent-range. In stable range the vortices decay by viscous diffusion; in irregular range the diffusion is turbulent. Crocco and Lees introduced the notion of interaction between viscous or dissipative flow near the surface of a body or in a wake, and the outer, plane, steady, nearly isentropic flow. Mixing or transport of momentum from outer stream to dissipative flow is the fundamental physical process.

BUFFETING AND SINGING OF PROPELLERS

Relation between turbulent wake phenomena and buffeting was discussed by Liepmann (see also paper by Luskin and Lapin, remarks by Krzywoblocki). Gutsche relates the singing of propellers in water with vortex street, and Gongwer shares his opinion. Literature on the latter problem has been reviewed by Work.

HYDRAULIC ANALOGY

Resemblance between waves formed by a body moving at supersonic speed through air and surface waves produced by a moving ship or water flowing past bridge piers was first commented on by Mach and developed by Jouquet and Riabouchinsky. Literature on this subject is extensive (not given here); some recent developments are discussed by Black and Mediratta.

EXPERIMENTAL METHODS IN WATER

Description of towing systems (body at rest, water is moving, or vice versa) is in the papers of Tietjens and in the book by Prandtl-Tietjens. Shadowgraph method was applied by Abramson. Rosenberg used colloidal solutions for visual-flow studies. Aluminum powder technique is well known and does not need to be described. Photographic technique sometimes gives quantita-

tive results. It was used by von Kármán, Rubach, Schmidtke, Krüger, Brown (smoke tunnel), Osborne and Morelli, Nicholson and Field (flame stabilization), and others. Progress in that technique in France is in Weyl's report. Pressure pickup is used in underwater ballistics. For ordinary tests it must be highly sensitive. Survey of magnetic induction method applied to measurements of fluid velocities is in Charwat's thesis. Electromagnetic recorder in low-speed wind tunnel was used by Krzywoblocki. Hot wire for frequency measurements was used by Tyler, for uniform flow measurements by Richardson and Middlebrook. Usually it is used for measuring fluctuation velocities. Hot-wire construction in water is in the paper by Macovsky and in Taylor Model Basin Report no. 726 (with amplifier described by Schubauer and Klebanoff). Tests show that wire is contaminated by gaseous and solid impurities in water. Analogous problem in air (dust) is treated by Collins. Thermistor (paper by Dowell) was used to measure mean velocity below four knots. Gongwer used sound technique to measure the frequency of singing vanes (cathode-ray oscilloscope and stationary microphone, standard sound analyzer, etc.).

NEW THEORY OF VORTEX STREETS

Recently, G. Birkhoff (1952 and 1953) put forward a new theory of vortex streets: The mean longitudinal spacing a and mean transverse spacing b are invariant. It follows from this and other known facts, that the periodicity of a vortex trail is unstable, not the ratio b/a . Birkhoff supported his theory by means of comparison of its results with previous investigations by Helmholtz (1868), Ahlborn (1902), Relf and Simmons (1921), Jeffreys (1930), Rosenhead (1931, 1932), Schiller and Linke (1931, 1933), Durand (1933), von Mises (1935), Camichel and Teissie-Solier (1935), Homann (1937), Schwarz (1937), Goldstein, Milne-Thomson (1938), Castagnetto (1939), Rosenhead (to be published), and many others.

More detailed review is contained in the author's Technical Memo. no. 1552, June 1953, issued by NOTS, China Lake, Calif.

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Theoretical and Experimental Methods

(See also Revs. 2683, 2765)

2669. Einstein, A., Generalization of gravitation theory, Reprint of Appendix II from the fourth edition of "The Meaning of Relativity," Princeton, N. J., Princeton Univ. Press, 133-165, 1953. \$3.50.

The fourth edition differs from the third only in the revision of Appendix II which contains, primarily, a modification of Einstein's unified field theory. The fundamental idea is still the replacement of the symmetrical tensor which represents the gravitational field in general relativity by a real asymmetric tensor g_{ik} , which represents the "unified" gravitational and electromagnetic field, and the introduction of the components Γ_{ik}^j of a real asymmetric connection as the additional field variables. Now, however, only systems of field equations derivable from a variational principle are considered, thus insuring the compatibility of the system. This forces abandonment of the system favored in the original Appendix II which contained more equations than variables. System (Ib) of the original Appendix II is proposed. The linear approximation of the field equations leads to two sets of equations, one each for the symmetric and antisymmetric field quantities. The latter set is asserted to "constitute a generalization of Maxwell's equations for empty space." Einstein has added a highly interesting discussion related to his philosophy of science. As he points out, the difficult and as yet unaccomplished task remains of confronting this theory with experiment, and until this is done its validity cannot logically be discounted. G. Salzman, USA

2670. Cohn, G. I., and Saltzberg, B., Solution of nonlinear differential equations by the reversion method, *J. appl. Phys.* **24**, 2, 180-186, Feb. 1953.

Authors demonstrate how reversion method reduces a common class of ordinary nonlinear differential equations to a set of ordinary linear equations which are identical except for recurrent driving functions expressed in terms of the driving functions of the original nonlinear differential equation. This characteristic is used to derive a "generating" expression for the general driving function in the auxiliary set. Example solutions by the reversion method for a few elementary equations, such as $u_x + u^2 = 1$, for which exact solutions are known, are graphically illustrated in termwise approximation to show that such solutions give reasonably rapid convergence.

P. J. Schneider, USA

2671. Salvadori, M. G., Extrapolation formulas in linear difference operators, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 15-18, 1952.

Author shows that, for many linear problems solvable by finite difference methods, improved results may be obtained from approximate solutions for two or more network mesh sizes by Richardson's extrapolations. Formulas and tables are presented to expedite these extrapolations for various ways in which the finite difference solution can converge to the correct result with decreasing mesh size. Several numerical examples are shown for which the error is reduced 75% or more.

P. Seide, USA

2672. Fritz, N. L., Analog computers for coordinate transformation, *Rev. sci. Instrum.* **23**, 12, 667-671, Dec. 1952.

Problems in multicomponent analysis frequently require repetitive solutions of a set of simultaneous equations in which the coefficient terms do not change. In these cases, it is advantageous

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to obtain a general solution from which the specific solutions may be obtained readily with simple instruments. An instrument is described which will solve a three-equation set as rapidly as an operator can set the values on the input dials, and will simultaneously present the three answers with an accuracy of better than 0.2% of maximum value. A method is presented for calculating the accuracy of computation for use in the design of instruments of this type.

From author's summary

2673. Morris, J., On the solution of linear simultaneous differential equations with constant coefficients by a process of isolation, *Aero. Res. Coun. Lond. Rep. Mem.* 2623, 7 pp., Sept. 1947, published 1952.

A method of solution is given as an alternative to the usual Heaviside method. Somewhat greater flexibility is achieved in the handling of the constants of integration.

S. A. Schaaf, USA

2674. Hornich, H., On the solutions of certain elliptic differential equations (in German), *J. reine angew. Math.* 189, 204-206, 1952.

Author considers the equation

$$a^2(r^2\partial^2 u/\partial r^2 + r\partial u/\partial r) + \partial^2 u/\partial \varphi^2 = f$$

where a is an irrational real number between 0 and 1,

$$f = \sum_{n=0}^{\infty} r^n \sum_{\substack{\nu=n, n-2, \dots \\ \nu \geq 0}} (\alpha_{\nu n} \cos \eta \varphi + \beta_{\nu n} \sin \eta \varphi)$$

($|\alpha_{\nu n}| \leq M$, $|\beta_{\nu n}| \leq M$ for all ν, n). It is shown that a necessary condition, that the equation have no twice differentiable solution near $r = 0$, is that a belong to the set of Liouville transcendental numbers for which $\liminf |a - \nu/n|^{1/n} = 0$ for all pairs of integers ν, n whose difference is even.

Courtesy of *Mathematical Reviews*

F. Browder, USA

2675. Newmark, N. M., Bounds and convergence of relaxation and iteration procedures, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 9-14, 1952.

This paper discusses a number of general concepts relating to relaxation procedures, methods of "steepest descent," and iteration procedures. The relationship between these is indicated for certain systematic ways in which relaxation patterns may be developed. An extrapolation procedure is developed for problems in which the successive-approximations technique, in its usual form, diverges. The procedure is a generalization of an observation by Hartree and others that, in many cases, such divergent problems can be treated by considering the divergence to be a geometric one. The study of upper and lower bounds to the errors in relaxation procedures is a second major part of the paper. General procedures are developed for determining the maximum and minimum errors in any of the quantities for which bounds to an influence function can be determined. Use of the theorems derived makes it possible, for example, to estimate the error due to all sources, including "round off" at any stage in the numerical solution of the Laplace or Poisson equation.

From author's summary by P. J. Schneider, USA

2676. Helmer, C. H., A vector method for solving vibration problems, *Engineering* 174, 4529, 4531; 620-621, 685-686; Nov. 14, 28, 1952.

A graphical method using vector diagrams is developed to solve certain vibration problems. The natural damped oscillations

and the forced oscillations of a system of one degree of freedom are considered, namely, a mass suspended from a rigid ceiling by means of a spring. Although the vector method is not new (a treatment can be found in Den Hartog's "Mechanical vibrations," for example), the author treats the above cases more fully and in a slightly different way. This method is useful in that it brings out the physical significance of the vibration problem at hand. It is also an aid to those who find the mathematical treatment difficult to understand.

To the reviewer, the value of any method lies in the possibility of its being extended to include more complicated problems. Since, as is pointed out by the author, the vector method can be extended to such problems as the response of relays, servomechanisms, etc., the method is well worth cultivating.

E. J. Scott, USA

2677. Lee, T. D., On some statistical properties of hydrodynamical and magneto-hydrodynamical fields, *Quart. appl. Math.* 10, 69-74, 1952.

The Fourier transforms of the standard equations of magneto-hydrodynamics [cf. Batchelor, *Proc. roy. Soc. Lond. (A)* 201, 405-416, 1950; and Chandrasekhar, *Proc. roy. Soc. Lond. (A)* 204, 207, 435-449, 301-306; 1951] are written down and the equivalent in the wave number (k) space of the law of conservation of energy which obtains in the absence of viscosity and conductivity is interpreted as meaning that the spectral law of the kinetic energy (\bar{u}^2) and the magnetic energy (\bar{H}^2) are both governed by the density of free modes which exist in the k -space, namely, $4\pi k^2 dk$. From a heuristic discussion of the case when dissipation by viscosity and conductivity takes place, the author concludes that, in the limit of large Reynolds numbers, both \bar{u}^2 and \bar{H}^2 follow the $k^{-3/2}$ -Kolmogoroff law. [The author's discussion ignores the fact that \bar{u}^2 admits a Loitsianskii invariant independently of the growth of a small initial magnetic field by the stretching of the lines of force, and that, therefore, there can be no transfer of energy from the velocity to the magnetic fields for $k \rightarrow 0$; from this last circumstance one can conclude that the spectrum of \bar{H}^2 for $k \rightarrow 0$ must be as k^4 in contrast to k^3 which obtains for \bar{u}^2 .]

Courtesy of *Mathematical Reviews*

S. Chandrasekhar, USA

2678. Vermeulen, R., Dimensional analysis, units and rationalization, *Philips Res. Rep.* 7, 6, 432-441, Dec. 1952.

Author shows by illustrative mechanical and electrical examples that indiscriminating manipulations of dimensional equations may result in such contradictions as $1 \text{ sec} = 3 \times 10^9 \text{ cm}$ or the like. Such possibilities are eliminated by a precise definition as to the meaning of "multiplication of physical quantities" and by forestalling a mingling of multiplications whose physical interpretations differ according to this definition. Efficacy of the suggested procedure is illustrated by several examples. The same approach eliminates certain common misinterpretations stemming from rationalization of a system of units.

T. J. Higgins, USA

2679. Pailloux, H., Some applications of function theory in rational mechanics (in French), *Ann. Sci. École Norm. Sup.* (3) 69, 213-257, 1952.

If f is a functional of a function x^P of variables P , and if the variation of x^P in the domain D gives a variation $\delta f = \int_D g_P \delta x^P d\tau$, where $d\tau$ is an element of D , then g_P is the functional derivative of f with respect to x^P , and is denoted in this paper by $\partial f / \partial x^P$. In this way, the equations of equilibrium of an elastic body, for example, can be written $\partial \bar{\omega} / \partial u = X$, $\partial \bar{\omega} / \partial v = Y$, $\partial \bar{\omega} / \partial w = Z$, where $\bar{\omega}$ is the energy, (u, v, w) the displacement, and (X, Y, Z)

the body force. Apart from the use of this notation, the paper is best described by saying that it consists in the application of variational methods to problems in the mechanics of continua. The problems treated include the motion of an inextensible thread on a plane and on a surface, the vibrations of an elastic rod of any form and section, and the equilibrium of a beam, nearly straight and of slowly varying section, under the action of body forces. In this last problem the method is approximate, the displacement being assumed to be linear in the two coordinates in the plane perpendicular to the axis of the beam. The nine coefficients in the expressions for the displacement are functions of the coordinate in the direction of the axis, and the variational condition gives nine linear differential equations for them. The result is a formula for the bending of a beam which differs from the classical formula $EI/\rho = -M$ by the addition of a term on the right proportional to the transverse load and by the change of E to $E(1 - \sigma)(1 + \sigma)^{-1}(1 - 2\sigma)^{-1}$, where σ is Poisson's ratio. (The functional derivative, as used in this paper, appears to be equivalent to the variational derivative [Th. de Donder, "Théorie invariante du calcul des variations," Gauthier-Villars, Paris, 1935, p. 8]. The variational derivative, usually written with δ instead of Δ , has the advantage of being explicitly defined, once for all. The author indeed at one point obtains this explicit form for a particular case, but makes no reference to the variational derivative by name. No references to literature are included in the paper.)

J. L. Synge, Ireland

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 2669, 2686, 2706, 2707)

2680. **Legger, R. J., The d'Alembert principle**, Anniv. Vol. appl. Mech., dedicated to C. B. Biezeno; Haarlem, Antwerpen, Djakarta, N. V. De Technische Uitgeverij H. Stam, 1953, pp. 121-131. Hfl. 20.

Author solves two kinetics problems with the aid of d'Alembert's principle and compares his solutions with those obtained by use of equations of motion. Each of the problems requires the determination of internal forces and moments at a section of a rotating rigid body. Author concludes that the use of the d'Alembert principle is much simpler than the use of equations of motion. Reviewer believes that author has not made the best selection of equations of motion. P. G. Jones, USA

2681. **Grammel, R., Nonlinear vibrations of an infinite number of degrees of freedom** (in German), Anniv. Vol. appl. Mech., dedicated to C. B. Biezeno; Haarlem, Antwerpen, Djakarta, N. V. De Technische Uitgeverij H. Stam, 1953, pp. 109-118. Hfl. 20.

Author considers two cases of free vibrations of elastic systems: torsional vibrations of circular cylinders, flexural vibrations of prismatical bars. Governing nonlinear partial-differential equations are derived, using stress relations which deviate from Hooke's law as suggested by Kauderer [AMR 3, Rev. 2219]

$$\sigma = E\epsilon(1 + \lambda\epsilon^2), \quad \tau = G\psi(1 + \lambda^1\psi^2)$$

where λ, λ^1 are constants of the material. Author then proceeds in each case to solve differential equation by taking solution to be the product of a space and a time function and then using the customary perturbation procedure. A few numerical values are given for the torsion problem, which is developed in more detail than is the flexural problem. Paper is very clearly written.

H. N. Abramson, USA

2682. **Yamada, K., Fundamental theory of toothed gearing**, I, II, III, IV, V, VI, VII, *Proc. Japan Acad.* 25, 84-89, 90-96, 97-102, 133-138, 139-144, 145-150, no. 10, 1-6, 1949.

The relative motion of two gears is obtainable by pure rolling (without sliding) of a curve on one gear over a curve on the other gear; the curves are called the pitch curves. The tooth profiles carried by the gears also roll on each other, but with sliding. In these papers, the well-known fundamental theorems of gearing are derived. They include the relations between the profile curves and the pitch curves, the derivation of the profile curves as roulettes of the pitch curves, the equations of the curves, and the radii of curvature. The general theorems are applied to the two most prevalent systems of gearing—the cycloidal system and the involute system—to determine the equations of the profile curves and the paths of contact.

The plane theory is extended to curves on the surface of a sphere where the great circles play the role of straight lines. The general geometrical theory is developed for the rolling of curves over curves. Analytical expressions in terms of spherical polar coordinates are derived. The spherical analog of the Euler-Savary formula for plane rolling curves is derived and used. Applications are made to the determination of the spherical cycloidal bevel gears (small circles), between involute bevel gears (great circles), and between octoid bevel gears (conchoid or figure-eight curves). Expressions for the specific slidings are derived for each case.

The work is carried out with much greater mathematical detail than is found in textbooks. However, the thoroughness of the derivations is marred by the complete lack of figures which would make these papers excellent textbook material.

Courtesy of Mathematical Reviews

M. Goldberg, USA

2683. **Levi-Civita, T., and Amaldi, U., Lectures on rational mechanics**. Vol. I Kinematics—principles and statics; Vol. II Dynamics of systems with a finite number of degrees of freedom Part I. [Lezioni di meccanica razionale. Vol. I Cinematica—principi e statica; Vol. II Dinamica dei sistemi con un numero finito di gradi di libertà, Parte prima], Bologna, Nicola Zanichelli, 1950, 1951; xviii + 816 pp., ix + 510 pp. L 5000; L 4000.

Vol. I of this valuable treatise, stemming from lectures given by the first author at the universities of Padua and Rome and by the second author at the universities of Modena and Padua, appeared in 1923, with a second edition in 1930. Parts I and II of vol. II appeared in 1926 and 1927, respectively. The present edition is a slightly revised and corrected version of the earlier one. It is published to honor the memory of its first author, Prof. T. Levi-Civita, who died on December 29, 1941, in Rome.

This treatise is one of the greatest and most excellent works on the subject. The mode of presentation of the material may be considered as a model for those writing textbooks on theoretical mechanics. Emphasis is placed not only on the theoretical aspects and on a rigorous mathematical treatment, but also on clarity of exposition and on a detailed and careful analysis of the physical picture underlying the problems under consideration. Formulas for their own sake do not constitute the final aim; a proper understanding of physical reality in terms of mathematical analysis is the aim throughout the work. This feature of the exposition is characteristic of the "Lezioni" and distinguishes them among the other treatises on rational mechanics. The text of the book is amplified by an abundance of exercises, the complements and references to original papers serving to open vistas into more advanced fields. There is also, scattered throughout the book, some biographical material on mathematicians and physicists whose works are of fundamental

importance for the foundations and development of classical mechanics. Unfortunately, there are some shortcomings in the presentation of the contributions of the Russian workers in this field. For example, dealing with the equations of nonholonomic systems in § 8, chap. V of vol. II, part I, no mention is made of the works of S. A. Chaplgin (1869–1942) who first obtained some forms of these equations in 1895 [see AMR 6, Rev. 716]. In chapter VI ("Stability and vibrations") of the same volume, the works of N. E. Zhukovskii (1847–1921) should be mentioned, and some space should be given to the exposition of the results of Lyapunov (1857–1918) on stability of motion [see AMR 4, Rev. 4070]. In the first edition, as in the present one, some of Lyapunov's theorems are stated without proof. This certainly prevents the reader from appreciating in full the importance of his contributions to the theory of stability of motion.

Except for some of the afore-mentioned minor deficiencies, this treatise can be highly recommended to mathematicians, physicists, and theoretically minded engineers who, in reviewer's opinion, will enjoy reading it.

The chapter headings of the two volumes are as follows. Vol. I; I Theory of vectors. II Kinematics of a particle. III Kinematics of rigid bodies. IV Relative motions with applications to rigid bodies. V Plane motion of rigid bodies. VI Kinematics of systems. VII Fundamental concepts and postulates of mechanics. VIII Derived concepts of mechanics. IX Statics of a particle. X Geometry of masses. XI Newtonian attraction. XII Principle of reaction. XIII Statics of solids. XIV Statics of constrained systems. XV Principle of virtual work and general statics. XVI Relative equilibrium. Vol. II, part I; I Dynamics of a particle moving in prescribed trajectory. II Dynamics of a free particle and of a particle moving on a prescribed surface. III Elementary concepts of celestial mechanics. IV Dynamic and kinetic characteristics of systems. V General theorems on motion of systems. Lagrange's equations. Non-holonomic systems. VI Stability and vibrations.

E. Leimanis, Canada

2684. Scholz, N., Wind-tunnel measurements on motorcycle models (in German), *ZVDI* 95, 1, 17–20, Jan. 1953.

Paper deals with design of streamlined covers for record-breaking motorcycles. Six component measurements were made on about 30 models to determine resultant aerodynamic force at different wind speeds. Effect of side wind was also studied. Streamlining was found to reduce wind resistance about 50%, but introduces lifting and nonstabilizing side forces. Yawing moment was reduced by fitting fin at rear.

Only aerodynamic forces are considered. Interaction of tire and aerodynamic forces have been treated by G. E. L. Walker [AMR 4, Rev. 3766].

S. Sjöström, Sweden

2685. Rázsó, I., Contribution to the theory of roll resistance of a transport wheel on a deformable surface (in Russian), *Acta Techn. Hung., Budapest* 1, 3, 261–276, 1951.

The problem of rolling of a wheel on soft ground differs from the same problem on roads and rails. Author analyzes the influences of the velocity, central angle of contact area, driving earth's volume, and wheel material.

The most important influence is that of the velocity, because the ground mass varies with the third power of velocity and the sine of the central angle, especially where the ground is very soft. In this case, when $v = 8$ km/hr, the loss of the power is 3 hp.

The friction between the wheel and the ground also augments the roll resistance. If the central angle is greater than the double angle of the friction, ploughing appears, and the reaction of the rolling increases suddenly. With the proper selection of wheel dimensions, this effect can be diminished.

The paper is based on Russian literature. The summary is in German.

D. Rašković, Yugoslavia

Gyroscopics, Governors, Servos

(See also Rev. 2820)

2686. Shanmugadhasan, S., Quantization of classical spin theory, *Canad. J. Phys.* 31, 1, 1–10, Jan. 1953.

In continuation of earlier investigations [*Proc. Camb. phil. Soc.* 37, 40, 1942; title source, 29, p. 593, 1951, 30, 226, 1952], author treats with Dirac's method [*Canad. J. Math.* 2, 129, 1950] the cases where the particle behaves like a pure gyroscope and like a symmetrical top. When there is an interacting field, it is assumed that the action of this field is given by the effective 4-vector potential—without precise specification, and therefore with no discussion of the dynamics of field variables. The orthogonality of the velocity to the tensor dual to spin tensor is imposed as an alternative constraint condition. Theory considered here then shows more interesting mathematical and physical features than spin theory quantized previously. Paper is devoted to the canonical formalism, and quantum formulation is completed with help of standard analogy rules. Detailed discussion of the obtained equations will appear in a later paper.

H. Bilharz, Germany

2687. Bubb, F. W., Linear noise smoothing and predicting filters, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 167–174, 1952.

The design of an optimum predictor filter which both extrapolates a message to a future time and separates it from corrupting noise has been solved by Kolmogoroff and Wiener. In the present paper the same problem is solved with the mechanical engineer in mind who dislikes Fourier integrals and complex function theory. Remaining completely in the real time domain, paper derives, instead of Wiener's integral equation for the best memory function of the filter, the equivalent system of linear algebraic equations for a set of ordinates of this function. The numerical solution of these equations is also discussed and compared to Wiener's method.

H. L. Oestreicher, USA

2688. Mersman, W. A., Evaluation of an integral occurring in servomechanism theory, *Pacific J. Math.* 2, 627–632, 1952.

The theory of the best design of linear servomechanisms, as described by R. S. Phillips [see James, Nichols, and Phillips, "Theory of servomechanisms," New York, McGraw-Hill, 1947; AMR 2, Rev. 1359], leads especially to the evaluation of the integral

$$I = \frac{1}{2\pi i} \int_{-\infty-i}^{\infty-i} \frac{g(x)dx}{h(x) \cdot h(-x)}$$

with $g(x) = \sum_{k=1}^n g_k x^{2(n-k)}$ and $h(x) = \sum_{k=0}^n a_k x^{n-k}$, $a_0 \neq 0$, a_k real. A procedure for evaluating the integral I and formulas for I with regard to special polynomials g and h of low degree have been given by Phillips. The author develops a general formula, representing I as a rational function of the coefficients a_k and g_k . The result is

$$I = \frac{(-1)^{n+1} \cdot ||g_{ij}||}{2a_0 \cdot ||c_{ij}||}$$

with $c_{ij} = a_{2i-j}$, $g_{ij} = g_i$ for $j = 1$, and $g_{ij} = c_{ij}$ for $j > 1$. It is to be understood that $a_r = 0$ for $r < 0$ and for $r > n$.

H. Bückner, Germany

Vibrations, Balancing

(See also Revs. 2676, 2683, 2703, 2704)

2689. Gutin, L. Ya., On the theory of steady oscillations of an elastic semispace (in Russian), *Zh. tekhn. Fiz.* **21**, 8, 792-906, Aug. 1951.

On the basis of the asymptotic expression of H. Lamb [*Phil. Trans. Roy. Soc. (A)* **203**, p. 1] for the displacements on the boundary of an elastic semispace, produced by a periodic normal force and a periodic center of dilatation on the boundary, author obtains similar expressions for the displacements of a Rayleigh wave at any point of the semispace, produced by horizontal and vertical forces, doublets, and centers of dilatation.

G. Herrmann, USA

2690. Smith, R. A., On an equation connected with the theory of triode oscillations, *Proc. Camb. phil. Soc.* **48**, 698-717, 1952.

The differential equation $\ddot{x} + k\dot{x}f(x) + x = pk\lambda \cos(\lambda t + \alpha)$ is investigated for existence, amplitude, and stability of periodic solutions when the parameter k is small. The method resembles that of M. L. Cartwright, "Contributions to the theory of non-linear oscillations" [AMR **4**, Rev. 555]. The principal aim of the author is to establish conditions on $f(x)$ which guarantee that the results obtained in the paper cited above for the case $f(x) = x^2$ remain qualitatively valid. The function $f(x)$ is assumed to satisfy the following conditions: (1) $f'(x)$ is continuous; (2) $f(x) \geq 0$; (3) $|f'(x)| \rightarrow \infty$ as $|x| \rightarrow \infty$; (4) $f(0) < 0$. The main criterion depends on properties of the function

$$\theta_1, \theta_2 = \frac{2}{\pi} \int_0^{\pi/2} \sin \tau \int_0^{b \sin \tau} \{f(x) + f(-x)\} dx d\tau$$

Let θ_1 and θ_2 be the positive roots of $\theta'(b)$ and $\theta(b)$, respectively, and assume that $b\theta'(b) \geq b\theta''(b)$. Then the number and nature of solutions of the differential equation having period $2\pi/\lambda$, with λ near 1, is shown to vary with λ in qualitatively the same way as in the case of Van der Pol's equation. This criterion is used to answer the special question as to how "flat" $f(x)$ could be without destroying the essential similarity to the case $f(x) = x^2$. The investigations are then extended to the subharmonics of order $1/2$.

Courtesy of Mathematical Reviews

W. Wasow, USA

2691. Ayre, R. S., Jacobsen, L. S., and Hsu, C. S., Transverse vibration of one- and of two-span beams under the action of a moving mass load, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 81-90, 1952.

Response of simply supported multispan beams under action of moving mass loads is investigated. Analytical results deduced in accordance with the classical methods of trigonometric series expansions are subjected to experimental verification. Excellent agreement between the results of numerical analysis and measurement techniques employed is established. Emphasis of study is on the experimental side. The underlying theory of transverse beam vibrations is the classical theory which, as is well known, does not account for rotatory inertia and shear effects. Of course, in a case like this one, where no rapid changes in the transverse response should be expected, the choice of this theory instead of the Timoshenko bending mechanism is well justified. Carefully elaborated diagrams on the experimental setup and the response characteristics illustrate the excellent contributions of the authors to the theory of beam-bending phenomena.

M. A. Dengler, USA

2692. Smith, F. C., and Howard, D. M., Calculation and measurement of normal modes of vibration of an aluminum-alloy box beam with and without large discontinuities, *NACA TN 2884*, 40 pp., Jan. 1953.

Authors describe a method of collocation for aluminum box beams of the type used in aircraft structures, including the effects of concentrated masses and large cutouts. A theoretical development based on the use of influence coefficients and an iterative solution is presented for the normal modes and frequencies of box beams.

Experiments were conducted to assess the accuracy of calculated results. It was found that root flexibility materially influences the accuracy of the natural frequency calculation and has a lesser effect on the normal modes. Authors claim that the collocation method is sufficiently accurate to calculate the fundamental and several higher modes and frequencies if the root flexibility is properly included in the analysis.

J. B. Duke, USA

2693. Wittmeyer, H., A simple method for approximate calculation of all torsional natural frequencies of a rod of variable cross section (in German), *Ing.-Arch.* **20**, 5, 331-336, 1952.

The eigenvalue problem of the torsional vibrations of a shaft with smoothly varying cross section is treated by a parameter method which not only closely approximates the lower frequencies but also has the proper asymptotic values at very high frequencies. By a suitable modification of the equivalent variation problem, a perturbation parameter is obtained which admits the determination of the lowest frequencies to within a relatively small error. The method is readily adapted to other vibration and buckling problems for their eigenvalues but not the corresponding eigenfunctions.

D. L. Holl, USA

2694. Myklestad, N. O., Numerical analysis of forced vibrations of beams, *J. appl. Mech.* **20**, 1, 53-56, Mar. 1953.

This is a straightforward application of modified Holzer's method for finding natural frequencies and modes as employed by author [*J. aero. Sci.* **11**, 153, 1944] and by others. Since exciting frequency is given, only one single set of calculations at that frequency is needed. Phase differences of exciting forces and of deflections as well as internal damping are taken care of by using standard complex representation.

L. S. Dzung, Switzerland

2695. Stanford, E. G., A contribution on the velocity of longitudinal elastic vibrations in cylindrical rods, and on the relationship between Young's modulus and temperature for aluminum, *Atti Conv. internaz. Ultracust.* **1950**, 332-340, 1951.

From a number of aluminum bars with dimensions ranging from $1/2$ -12-in. diam and 10-20-in. length, the natural modes of longitudinal vibrations and their velocities were determined. With the experimental setup, the rods were clamped in the midsection, one end was connected to a magnetostrictive exciter, the other end bore a magnetostrictive pickup. The exciter was driven by a variable-frequency oscillator and the natural frequencies were determined by an output meter in the pickup amplifier.

As a relationship exists between the velocity of the longitudinal waves and Young's modulus, it was possible to determine the influence of temperature on this modulus in the range from 0-600°C by slowly treating the test bar in a furnace and taking measurements at short intervals. The reading of the output meter at the natural frequency is a measure of the damping of the system, and so the relative damping values as a function of temperature could be found. Although the investigations were not

finished at the publication of the paper, some interesting results have already been obtained, such as a dip in the Young's modulus curve at about 500 C and a sudden rise in the damping value at the same temperature, both being due to recrystallization effect.

R. G. Boiten, Holland

2696. Wu, M. H. L., Subharmonic resonance of system having non-linear spring with variable coefficient, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 147-153, 1952.

Author investigates subharmonics of differential equation $m\ddot{x} + c\dot{x} + F(x, \Omega_k, t) = 0$ [1]; here m, c, Ω_k are constants. A variable change $\dot{x} = u(x)$, commonly used for autonomous systems, converts [1] into two nonlinear integrodifferential equations for time and velocity. An iteration process is then set up and is applied to a system in which $F = [1 + p \cos(n\omega t + \phi)]x^3$. Numerical calculations are made for third- and fourth-order subharmonic resonances. The iteration is tedious. Convergence is not discussed. Existence and stability of subharmonics are taken for granted, for which reference may be made to Chike Obi [*Proc. Camb. phil. Soc.* **47**, 741-751] and Levinson, N. [*Annals Math.* **45**, 723-37, 1944]. For further bibliography, see also Cartwright, M. L., Copson, E. T., and Grieg, J., *The Advancement of Science*, **6**, 21, 1-12, 1949. The paper may be useful to an engineer.

A. C. Eringen, USA

2697. Gordon, D. S., Instruments and techniques in vibration research, Trans. Instn. Engrs. Shipb. Scotland **96, 3, 71-124, 1952-53.**

Methods of producing and measuring vibrations are discussed generally, particular mention being made of equipment developed for a research project involving the vibration testing of ship models up to 20 ft in length, in air and in water. Special items developed for this project include small vibration exciters with high stability of frequency and force, lightweight detectors, and a phase-measuring unit. A critical analysis of different systems is made and their performances are compared.

C. E. Crede, USA

2698. Woodward, J. G., A vibrating-plate viscometer, J. acoust. Soc. Amer. **25, 1, 147-151, Jan. 1953.**

This type of viscometer is simple to operate and well suited for rapid measurements of Newtonian viscosity and for empirical indications of flow changes of some non-Newtonian materials. A circular plate (0.2-in. diam and 0.010 in. thick) is attached to a vibrating reed. The plate-reed system is operated at its resonance frequency of about 800 cps, so that the amplitude of vibration is a measure of damping, for a constant driving force. Two barium-titanate slabs are fastened to the reed near its support and generate a voltage which is proportional to the amplitude of vibration. Thus the reduction in voltage from that obtained in air when immersing the vibrating plate in a liquid is a measure of liquid damping. Author derives a theoretical relation between viscosity, density, and voltage change and demonstrates its validity by experimental data. Author's theory assumes that the resonance frequency of the reed-plate system in the liquid is equal to that in air, but author realizes that this assumption might not be applicable for materials of higher viscosity than about 10 poises. To obtain more significant measurements for heavier materials, a second disk (0.032 in. thick) is provided which gives the system a resonance frequency of about 400 cps. However, the applicability of the measurement to the exact interpretation of flow data of heavier liquids and of non-Newtonian materials is doubted, especially since end effects due to the finite thickness of the plate might exist. Nevertheless,

reviewer believes that this type of viscometer will find extensive use, especially in production control, since its electronic features provide advantages in adapting it to continuous operational processes.

Ruth N. Weltman, USA

Wave Motion, Impact

(See also Revs. 2691, 2723, 2956)

2699. Newlands, Margery, The disturbance due to a line source in a semi-infinite elastic medium with a single surface layer, Phil. Trans. roy. Soc. Lond. (A) **245, 896, 213-308, Dec. 1952.**

In order to improve and check the ray technique in geodynamics, the problem as formulated in the title is solved rigorously. This is done by the usual mathematical methods of Sommerfeld-Weyl and Bromwich, which obtain the solution at first as a complex (double) integral and then transform it by distortions of the paths of integrations into a form suitable for physical interpretation. For a final evaluation the method of steepest descent is used. The results show that at remote points, at or near the surface, pulses are received which correspond to the ray method, but there are also additional pulses, due to diffraction, which are not obtained by ray methods. For points in great distance this description of the disturbance is, though correct, no longer advantageous because the various pulses interfere with each other. Author shows, after a study of the free waves near the surface, that by an appropriate distortion of the path of integration the disturbance at great distances is essentially a Rayleigh wave train.

Many numerical results in the form of curves are presented and the application to seismological problems is discussed.

H. L. Oestreich, USA

2700. van den Dungen, F. H., Formulas for the numerical integration of the wave equation (in French), Acad. roy. Belgique, Bull. Cl. Sci. (5) **38, 7, 669-684, July 1952.**

Author resumes his efforts from earlier work [title source, (5) **38**, 1-2, 39-49, Feb. 1952] to adapt method of characteristics to numerical solutions to be carried out in a digital machine. Present communication gives wave function in terms of more symmetric linear expressions in values of initially given functions within the characteristic cone for a finite number of time intervals. Formulas given apply to two-dimensional waves. Extension to three dimensions is discussed briefly.

F. K. G. Odqvist, Sweden

2701. Birkhoff, G., and Kotik, J., Fourier analysis of wave trains, "Gravity waves," Nat. Bur. Stands. Circ. **521, 221-234, 1952. \$1.75.**

Various difficulties in defining a satisfactory general class of one-dimensional trains of gravity waves are pointed out. Formal Fourier transform theory is used to suggest plausible existence and uniqueness theorems. In particular, the following three conditions on a wave train are shown to be formally equivalent: The spectra of the components traveling in opposite directions are nonoverlapping; the total kinetic energy in space is independent of time; the total kinetic energy (density) observed at a point, totalled over time, is independent of the location of the point in space.

Finally, a Fourier analysis is made of the relation between elevation and pressure.

From authors' summary by V. P. Zimnoch, USA

2702. Fuchs, R. A., On the theory of short-crested oscillatory waves, "Gravity waves," *Nat. Bur. Stands. Circ.* 521, 187-200, 1952. \$1.75.

Wave systems on the surface of a liquid, which are periodic in both the direction of propagation and in the crest direction, called short-crested waves, are investigated to a second approximation. The surface profile, which was doubly sinusoidal on the first approximation, has broader troughs and narrower crests on the second approximation. It is found that for a mixture of long- and short-crested waves moving shoreward, the long-crested wave becomes more predominant as the water becomes shallower. It is also found that, in addition to an oscillatory motion, the particles have a constant velocity in the direction of the wave motion which is a maximum for lines passing through crests and troughs.

The linearized theory is applied to the case of the wave motion generated by an initially localized displacement.

L. Landweber, USA

2703. Dengler, M. A., and Goland, M., Transverse impact of long beams, including rotatory inertia and shear effects, *Proc. First U. S. nat. Congress appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 569-577, 1952.

Paper presents the first (correct) exact solution of a problem of transverse beam impact based on Timoshenko's differential equation (which accounts for the first-order effects of transverse shear and rotary inertia). Timoshenko's equation predicts the occurrence of two modes of wave transmission with, in general, different limiting velocities for the case of very short wave lengths; one velocity corresponds to a traveling shear jump, the other to a bending moment jump. For the special case where the beam properties are such that these two velocities coincide, authors derive, by application of Laplace and Fourier transforms, a closed expression for the time and space variation of the moment in an infinitely long uniform beam subjected to an impulsive point loading. Previous work by Uflyand [AMR 3, Rev. 33] on a similar problem is shown to be in error because of an improper choice of boundary conditions.

Reviewer's note: Uflyand's error has been made many times; it consists of applying the condition of zero slope at a clamped end or at an axis of symmetry even when the basic theory permits shear deformations. The appropriate condition is that of zero rotation of a transverse section.)

Authors also attack the more difficult case of a beam having unequal propagation velocities of shear and moment jumps. Laplace transforms involved apparently defy analytical inversion, but by application of contour integration, problem is reduced to several definite improper integrals, the numerical evaluation of which is not undertaken in the paper. B. Budiansky, USA

2704. Leonard, R. W., and Budiansky, B., On traveling waves in beams, *NACA TN* 2874, 76 pp., Jan. 1953.

The fundamental equations of the Timoshenko mechanism for the propagation of transverse waves in beams are analyzed in view of the development of efficient mathematical procedures of solution adequate to the ever-increasing importance of impulsive loading problems in aircraft structures and other fields of immediate application. Three principally different methods are employed: (a) The method of characteristics, which offers the advantage of a clear and simple geometrical interpretation of the inherent mathematical properties of the differential equations, and permits the establishment of a numerical step-by-step integration procedure; (b) the classical normal mode approach, which aims at the determination of the natural frequencies and modes of the beam in view of final representation of the solution

as a superposition of these elementary modal terms; (c) the derivation of exact solutions directly describing the characteristics of traveling-wave phenomena. Authors emphasize that exact traveling-wave solutions encounter serious mathematical difficulties and have, accordingly, been obtained only for a rather limited number of problems. Hence, while exact solutions might be considered as of superior logical value, approximative solutions of the type (a) or (b) will frequently be preferred from the standpoint of practical applications. Comparing methods (a) and (b), one is inclined to give preference to method (a) which is a true traveling-wave approach and has the advantage of relative numerical simplicity. There are two sets of characteristics corresponding to two propagation velocities c_1 and c_2 . The discontinuities of the bending moment M and the angular velocity ω of the beam cross section propagate along the one set, the shear force V and the linear velocity v propagate along the other set. Method (b) is characterized by the fact that a natural mode is now represented by a pair of functions instead of a single one, as in the elementary theory. A number of interesting examples are evaluated for the idealized case that the beam characteristics are such that the two wave-propagation velocities c_1 and c_2 become identical. Excellent diagrams demonstrating the convergence of the numerical procedures and the agreement between methods (a), (b), and (c) are included. The exact solutions are obtained by means of the Laplace transform. These latter are in absolute agreement with solutions obtained by M. Goland and the reviewer (see preceding review) in view of the well-known theorem that response to unit impulse corresponds to the derivative of response to unit step.

M. A. Dengler, USA

2705. Campbell, J. D., An investigation of the plastic behavior of metal rods subjected to longitudinal impact, *J. Mech. Phys. Solids* 1, 2, 113-123, Jan. 1953.

A method of impact loading is described which gives data regarding the onset of plastic deformation in a metal cylinder subjected to high-speed axial compression. The result is expressed as a dynamic stress-strain curve. This curve is constructed from wire strain-gage measurements in the rod during and after each of a sequence of longitudinal impacts. For each loading, the total (elastic plus plastic) deformation is computed as the maximum dynamic strain, measured during the loading, plus all previous permanent set, measured statically after each loading. The corresponding stress point is computed by multiplying Young's modulus by the difference between the dynamic strain and the residual permanent set for the particular impact. The results of tests on a soft aluminum alloy show that the average strain rate produced is about 15 per sec; the dynamic stress-strain curve is similar to the static curve but raised some 15-20%. W. R. Campbell [AMR 6, Rev. 1826] has made similar measurements on copper rods. His method of analysis differs, however, in the use of the plastic-wave velocity to compute stress.

J. M. Krafft, USA

2706. Beer, F. P., A method for the graphical determination of the motion of a structure with one degree of freedom under blast loading, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 175-178, 1952.

Author describes a graphical procedure for obtaining a solution to the equation $M\ddot{x} = P(t) - R(x)$, where M is the mass of a particle with one degree of freedom, \ddot{x} is the acceleration of the particle, $P(t)$ is an applied force which is any function of time, and $R(x)$ is a restoring force which is a function of the position of the particle. Method of solution is presented for $R(x)$ equal to a constant, a linear function of x , or a nonlinear function of x . $P(t)$ is any function of time including blast or impact loading.

W. B. Stiles, USA

2707. Plantema, F. J., Forces on elastic airplanes under landing impact (in Dutch), *Nat. LuchtLab. Rep.* S.373, 8 pp., 5 tables, 5 figs., Mar. 1952.

Author does not consider (as the title suggests) the elasticity of the air frame, but only the fore-and-aft bending deformation of the landing gear. This deformation is caused by the drag which acts on the wheel during the spin-up period. The equations are set up and solved for the prestress phase (before the oleo strut comes in action), for the remainder of the spin-up phase and for the ensuing spring-back motion. The influence of the metering pin is not considered in these equations. A numerical example is worked, and it is pointed out that the elasticity of the landing gear has a beneficial effect on the peak load. W. Flüge, USA

Elasticity Theory

(See also Revs. 2703, 2704, 2729, 2732, 2743, 2745, 2761)

2708. Novozhilov, V. V., Foundations of the nonlinear theory of elasticity, Rochester, N. Y., Graylock Press, 1953, vi + 233 pp., \$4.

This is an English translation of the valuable book by V. V. Novozhilov on the fundamental concepts of linear elasticity, a comprehensive review by H. I. Ansoff having been published in AMR 4, Rev. 3795. M. A. Dengler, USA

2709. Rivlin, R. S., The solution of problems in second order elasticity theory, *J. rational Mech. Analysis* 2, 1, 53-81, 1953.

Boundary-value problems in second-order elasticity theory according to E. D. Murnaghan [*Amer. J. Math.* 59, 235-260, 1937] may be solved systematically by the following process. Corresponding boundary-value problem in classical elasticity theory is first solved, neglecting second-order terms. From second-order theory, the additional forces are calculated which would have to be applied in order to maintain corresponding system of displacements. Additional force system is reversed in direction, and the displacements which it would then produce according to classical elasticity theory are calculated and added to the previous displacements, the sum thus giving the required solution. Method of solution is applied to a circular cylindrical tube subjected to simultaneous extension and torsion, and difference from classical results noted. Method also enables calculation of average effects throughout the body in cases when a detailed solution would be too laborious; e.g., simultaneous extension and torsion of a rod of arbitrary cross section. Modification of theory for incompressible material is also discussed.

F. K. G. Odqvist, Sweden

2710. Cattaneo, C., On the torsion of two elastic spheres in contact (in Italian), *Ann. Scu. norm. sup. Pisa* (3) 4, 1/2, 1-16, 1952.

Author formulates, in terms of an integral equation, the problem, solved previously by J. L. Lubkin [AMR 4, Rev. 4402] by means of potential theory, of two like elastic spheres in Hertz contact subjected to a monotonically increasing torsional couple. He obtains one- and two-term approximations to the distribution of the shearing stress on the contact surface, as well as to the relations between the radius of the slip annulus and the angle of twist and twisting moment, respectively. The reviewer notes that, while the stated expression for twist in terms of the slip radius consists of the first terms of a series expansion, for small applied moment, of the exact equation given by Lubkin, the approximate radius-torque relation does not check, even in the vicinity of zero torque, with the numerical values tabulated from the exact solution by Lubkin. In addition, the initial torsional

stiffness, which may be obtained directly from the linear approximation to the torque-twist relation, does not agree with that given by Mindlin [AMR 3, Rev. 2592] and by Lubkin.

H. Deresiewicz, USA

2711. Sternberg, E., Eubanks, R. A., and Sadowsky, M. A., On the axisymmetric problem of elasticity theory for a region bounded by two concentric spheres, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 209-215, 1952.

Classical boundary-value problem of finding stresses and displacements in a spherical shell under given arbitrary axisymmetric surface tractions is reinvestigated on the basis of stress-function approach of Boussinesq, referred to spherical coordinates ["Applications des potentiels" by J. Boussinesq, 1885]. The solution is presented in exact, explicit series form, of which the convergence has been examined, and is deemed to be more suitable for numerical computation. The cases of solid sphere and spherical cavity in an infinite medium have been deduced via a limit process. There is a bibliography at the end of the paper. D. N. Mitra, India

2712. Durelli, A. J., and Tsao, C. H., Discussion of the paper "Quantitative evaluation of residual stresses by the stresscoat drilling technique," *Proc. Soc. exp. Stress Anal.* 10, 1, 237-242, 1952.

On the basis of Kirsch's formulas for the stresses around a hole drilled in a plate subjected to hydrostatic or uniaxial tension (or compression), and assuming the maximum-stress law of failure of the Stresscoat, the discussers show that the amount of residual stress (or residual strain) is approximately proportional to the square of the distance between the center of the hole and the beginning of the crazing zone of the coating. In the original paper, the residual stress was assumed to be proportional to the distance between the edge of the hole and the beginning of the crazing zone, as previously claimed by G. Ellis. The discussers prove that the experimental data are in better agreement with their analysis. J. L. Serafin, Portugal

2713. Howard, H. B., Tube of least weight for given torsional stiffness, *J. roy. aero. Soc.* 57, 505, 45-46, Jan. 1953.

Using the Batho-Bredt theory, together with the calculus of variations, it is shown that the thickness of the tube should vary along the length, directly as the square root of the torque and inversely as the enclosed area. To increase the thickness of a tube to provide for a given increase in torsional stiffness, the increased thickness must also obey this law, which also holds for tubes of variable thickness, provided that the thickness variation around the perimeter is constant for all sections. Practical suggestions are made for the stiffening of a tapered tube under concentrated torque at the free end, such as box beams in wings and rear fuselage bodies. H. D. Conway, USA

2714. Southwell, R. V., and Vaisey, G., A problem suggested by Saint-Venant's "Mémoire sur la torsion des prismes," *Anniv. Vol. appl. Mech.*, dedicated to C. B. Biezeno; Haarlem, Antwerpen, Djakarta, N. V. De Technische Uitgeverij H. Stam, 1953, pp. 100-106. 20 Ill.

In Saint Venant's solution (1855) of the torsion of a rectangular bar $2a \times 2b$, contour-type diagrams were given for a square section and for a rectangular section with $a = 2b$. The first had 8 areas of symmetry, the latter, 4 areas. Saint Venant calculated the limiting value between these two conditions as $a = 1.4513 b$. The authors present similar diagrams for 5 ratios intermediate between $a/b = 1$ and 1.4513, showing how 4 regions

at of the 8 are squeezed out more and more as the upper limit is approached. Relaxational methods were used in the analysis.

M. W. Jackson, USA

2715. Hiedemann, E., and Spence, R. D., On a uniform theory of relaxation phenomena (in German), *Z. Phys.* **133**, 109-123, 1952.

The authors attempt to unify and generalize the mathematical side of existing relaxation theories. Any use of the specific laws of mechanics is avoided. The first part, by the first author, discusses the philosophy of the subject. The second analytical part is by the second author. He considers the equation $E = KX$, where X and E stand for any complex stress and the accompanying strain, both depending on time as $e^{i\omega t}$, and where K is a "complex modulus of elasticity," a function of ω . He replaces $E = KX$ by a corresponding complex Fourier integral representation of $E(t)$ in terms of X . He next assumes that if $X = 0$ for $t < t_0$, then also $E = 0$ for $t < t_0$, and conversely; and that if X is real, so also is E , and conversely. He then uses function-theoretic methods to discuss the consequences of these and other restrictions on the behavior of $E(t)$. Thus he obtains various types of "relaxation spectra."

C. Truesdell, USA

2716. Grioli, G., Validity of the theorems of Menabrea and the integration of the problems of nonisothermal elastostatics (in Italian), *R. C. Semin. mat. Univ. Padova* **21**, 202-208, 1952.

The author extends the theorem of Menabrea to certain types of linear thermoelastic deformations. Then he uses an extension of the method of the paper by Signorini [AMR **6**, Rev. 768] to obtain a lower bound for the temperature change in such a deformation.

C. Truesdell, USA

2717. Yu, Y.-Y., Solution for the exterior of a general ovaloid under arbitrary loading and its application to square, rigid core problems, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 227-237, 1952.

By means of the complex variable method of Muskhelishvili for plane stress problems, author has shown in an earlier paper the solution for the exterior of a general ovaloid under arbitrary loading, with constant or zero forces at infinity. In the present paper, a rigid core of the shape of a square with rounded corners is in a tension field of arbitrary inclination; secondly, in a beam subjected to pure bending; thirdly, the rigid core is subjected to arbitrary forces. The last case includes three special cases, a force at the center with two components and a couple. Numerical calculations and stress graphs clarify the results.

H. Neuber, Germany

2718. Drucker, D. C., Greenberg, H. J., Lee, E. H., and Prager, W., On plastic-rigid solutions and limit design theorems for elastic-plastic bodies, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 533-538, 1952.

The theorems recently developed by Drucker, Greenberg, and Prager determine upper and lower bounds for the collapse or limit load of a supported elastic-plastic body subjected to external loads. Recent work on complete solutions of plane strain boundary-value problems for plastic-rigid bodies has determined some stress and velocity fields which satisfy the laws of plasticity and equilibrium. If the limit-load theorems are applied to such complete solutions, the upper and lower bounds coincide, and so the exact limit load for the elastic-plastic problem is obtained.

Examples of such solutions are given, including problems for

which, because of the general form of the boundaries, it would be virtually impossible to attempt the elastic-plastic problem. Incorrect solutions, which satisfy all conditions in the plastic regions only, are also discussed.

From authors' summary by P. G. Hodge, Jr., USA

2719. Bölcskei, E., Deformation of thin membranes (in French), *Acta Techn. Hung., Budapest* **5**, 4, 489-505, 1952.

Mathematical paper derives three differential equations in the three displacements, which can be determined therefrom if the stress distribution in the membrane or the strain distribution due to temperature influences is known. The three relations are general and are expressed in rectangular coordinates. Author gives two applications of his theory: displacements of a cylindrical shell and of a plane membrane under its own weight.

M. Kuipers, Holland

2720. Strub, R. A., Distribution of mechanical and thermal stresses in multilayer cylinders, *Trans. ASME* **75**, 1, 73-79, Jan. 1953.

The design of thick-walled cylindrical pressure vessels, according to elastic stress theory, results in relatively heavy vessels if it is attempted to limit the high tangential stress at the inner cylindrical wall. It has been shown by tests by several investigators that a certain amount of plastic deformation takes place resulting in a redistribution of stress through plastic flow and corresponding prestressing of the inner layers of the cylinder when the applied pressure is relieved. If a perfect material were used in the design, it would deform in such a way that, when subjected to the maximum pressure load, the effective stress would be the same throughout. An approach to this ideal is the multilayer cylinder reported in this paper. The author gives several formulas by which mechanical and thermal stresses in such structures can be computed. A method is outlined with graphical aids by which the best proportions for this type of vessel may be determined. Optimum conditions are based on an effective stress derived from the maximum strain energy strength theory.

E. G. Allen, USA

2721. Davin, M., On the theory of stability and the intrinsic curve (in French), *C. R. Acad. Sci. Paris* **235**, 14, 691-693, Oct. 1952.

Cauchy's hypothesis, explaining cohesion phenomena by radial interaction, leads to particular forms of the elastic potential. If the theory of stability is applied to simple forms of potential obtained hereby, there result in general most of the phenomena that distinguish deformation and failure of certain engineering materials, principally neat cement pastes and mortars.

From author's summary by E. D. Fliess, Argentina

Experimental Stress Analysis

(See also Revs. 2697, 2769, 2779)

2722. Rondeel, J. H., On measurement methods used by the National Aeronautical Research Institute (N.L.L.) for the testing of structures (in Dutch), *Ingenieur* **65**, 10, L.1-L.6, Mar. 6, 1953.

Author describes the use of light in some structural tests to measure displacements and shearing angles of flat plates loaded in their plane, and in instruments as a means to obtain elliptically enlarged diagrams, and as an absolute standard to calibrate mechanical extensometers.

From author's summary by M. Botman, Holland

2723. Petersson, S., Investigation of stress waves in cylindrical steel bars by means of wire strain gauges, *Trans. roy. Inst. Technol., Stockholm* no. 62, 22 pp., 1953.

Author describes the use of strain gages to investigate the pressure pulse produced by an explosive charge detonated at one end of a steel bar. The uneven distribution of strain over the cross section of the bar with these short pulses due to radial inertia effects is allowed for by the use of two gages mounted at right angles to each other and used as two arms of a bridge circuit. The method described follows the classical work of Hopkinson [*Phil. Trans. (A)* 213, 437, 1914] and is similar to that of Davies [*Phil. Trans. (A)* 240, 375, 1948] except that strain gages are here used instead of condenser microphones. The novel results relate to the damping of the pulse by the bar, and author shows that high frequency components are damped out after having traversed the length of the bar (1 meter) several times.

H. Kolsky, England

2724. Kuske, A., Evaluation of experiments in two-dimensional photoelasticity (in German), *Forsch. Geb. Ing.-Wes. (B)* 18, 4, 113-126, 1952.

Author gives a critical report on experience with the different methods of deriving stresses from plane photoelastic observations. A new method, using the lines of constant shear stress, seems to offer some advantages.

H. Neuber, Germany

2725. Leven, M. M., and Frocht, M. M., Stress-concentration factors for single notch in flat bar in pure and central bending, *J. appl. Mech.* 19, 4, 560-561, Dec. 1952.

Stress-concentration factors, determined photoelastically, are given for notch-bend specimens. Variables are depth, root radius, and angle of notch for pure bending, and span-to-depth ratio for keyhole and Izod impact specimens. Information is given in form of four very clear and comprehensive diagrams.

R. Week, England

2726. Dodd, R. A., Residual stresses in aluminium alloy sand castings, *J. Inst. Metals* 81, 77-81, 1952/1953.

Residual stresses in aluminum castings were measured to determine effect of moisture in the sand mold, the pouring temperature, time between pouring and stripping, and the mold strength. A framework-type casting with different size members was cast, wire strain gages were attached, and then the member sawed from the framework. The results showed that: (1) The stresses increase with the moisture content, but over the usual range of 5% to 6.5% the difference was small; (2) the stress increases rapidly relative to the stripping time up to 10 minutes, but then the rate decreases until the maximum stress is reached in 1 hour; (3) the stress increases slightly as the pouring temperature increases; (4) the mold strength does not affect the stresses. The most practical observation is that the residual stresses will be minimized by stripping the castings as soon as possible.

C. R. Freberg, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 2695, 2751, 2776)

2727. Thomson, W. T., A note on tabular methods for flexural vibrations, *J. aero. Sci.* 20, 1, 62-64, Jan. 1953.

Presented method is applied to beam vibrations where stiffness and mass distribution are nonuniform. Generally, the exact mathematical procedure for such problems requires the establishment of differential equations which cannot be solved by elementary means. Instead of differential equations, method

uses a system of algebraic equations where deflection, slope, moment, and shear are varied finitely. This formulation suggests a successive computation by means of tables. Computation is started at one end of the beam where the boundary values are determined. Progressing from point to point, the quantities are obtained according to their form. Specifying the boundary conditions at another suitable point, the independent quantity is determined and, after that, all other quantities. Uncoupled flexural vibration, the same for a rotating beam, and coupled flexure-torsion vibration are treated in the note.

P.-P. Heusinger, Germany

2728. Swida, W., On the statics of annular beams (in German), *Beton u. Stahlbetonbau* 48, 1, 5-11, Jan. 1953.

Author tries to solve the problem of a horizontal circular ring vertically loaded and supported in n points. Reviewer regrets that he cannot agree with author's deductions nor with his results.

C. B. Biezeno, Holland

2729. Goldberg, J. E., On the application of trigonometric series to the twisting of I-type beams, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 281-289, 1952.

The resistance to twisting of I-type beams which are free to warp at the ends and are subjected to one or more torsional loads applied at intermediate locations derives primarily from (1) the Saint Venant type of torsional resistance, and (2) the resistance of the flanges to bending in their own planes. Representing the angle of twist as a sine series, the strain energy associated with (1) and (2) and the external work are calculated in terms of the Fourier coefficients which are determined from the theorem of virtual work. The resulting series may be differentiated term by term to evaluate the derivatives which characterize the stress distributions. For other end conditions, a residual solution may be superimposed which liquidates the untenable portions of the warping displacements.

From author's summary by G. Winter, USA

2730. Hoff, N. J., Complementary energy analysis of the failing load of a clamped beam, *J. appl. Mech.* 19, 4, 563-564, Dec. 1952.

A clamped beam is subjected to a uniformly distributed load which increases from zero to a value, causing the yield stress to be exceeded at the two ends and the center. Beam section is an idealized I and stress-strain curve, a straight line with slope E from zero stress to yield stress, followed by another straight line of slope E/C to fracture.

Use is made of an expression for total complementary energy to justify the fundamental assumptions of limit analysis. For chosen beam dimensions and $C = 290$, curves are given for the spreading of plastic regions and the variation of significant moments and deflections with increasing load. These curves show (a) that the conventional assumption of a yield hinge which permits rotation for small changes in moment below a critical load, is valid, and (b) that this critical load coincides with the formation of the third yield hinge (at center).

M. C. Steele, Scotland

2731. Heilig, R., The compound beam with arbitrary open cross section (in German), *Stahlbau* 21, 10, 186-189, Oct. 1952.

A theory is explained for the study of compound beams (steel and reinforced concrete) under the action of external forces and nonuniform heating. The elasticity equations obtained present an interesting analogy with those corresponding to homogeneous material. The effects of shrinkage and creep are not analyzed, although prestressing is considered.

I. Wolff, Brazil

Plates, Disks, Shells, Membranes

(See also Revs. 2711, 2735)

2732. Boresi, A. P., Coefficients of irregularity of a rotating system considering torsional elasticity of the shaft, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 111-117, 1952.

The coefficient of irregularity δ , defined as the percentage variation of angular velocity with respect to the mean angular velocity, is usually computed under assumption that the connecting shafts are rigid.

By means of a tabular procedure, part of which are complex Holzer tables, author shows that torsional elasticity can be taken into account. A typical example shows that the increased amount of labor is justified. δ varies with the place where it is determined. In the example studied, δ is 4 to 22 times the δ when flexibility is neglected.

W. L. Esmeijer, Holland

2733. Mansfield, E. H., Torsional stresses in multi-webbed rectangular cylinders, *Aircr. Engng.* 25, 287, 20-21, Jan. 1953.

Determination of the torsional stiffness and shear stresses of a multicelled rectangular cylinder is approximated by replacing the internal webs by an equivalent uniform web.

M. V. Barton, USA

2734. Czitary, E., and Heinrich, G., Safety analysis concerning slipping of a carrying cable off the mast (in German), *Ing.-Arch.* 6, 5, 372-386, 1952.

Besides vertical deflection, the carrying cable of an aerial cableway may have lateral deflection due to side wind force. Paper discusses how large side deflection may be if cable is not to fall off flat shoe grooves of passenger cableway supports. Influence of shoe friction and of flexural stiffness of cable which increase safety against falling off is not taken into consideration. In second part, stability of equilibrium of cable is discussed. Establishing a potential function for cable tension and wind pressure, authors derive figure of equilibrium from first variation, domain of stability from second variation.

From authors' summary by J. N. Goodier, USA

2735. Chu, C., A theory of twisted Bourdon tubes, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 271-280, 1952.

The Bourdon gages employed during the past hundred years were almost exclusively of the bent-tube type. The twisted-tube type was discovered at the same time, but was ignored for a long time. It is the purpose of this paper to establish a general theory of the twisted Bourdon tube, assuming the validity of the Saint Venant theory of torsion. The internal pressure causes a swelling of the tube cross section toward a circular form, which produces a torsional moment if no untwisting is permitted, or an untwisting if the torque is released. The case of a Bourdon tube with infinitesimal initial twist is first considered. The result, that torque at zero untwisting and angle of untwisting at zero torque are linear functions of the twist angle, no longer holds for tubes with finite initial twist, where both functions reach a maximum (critical and optimal initial twist). The last part of the paper deals with the case of finite initial twist and finite untwisting angle, the untwisting angle being no longer a linear function of pressure.

The general theory is applied to tubes of narrow rectangular cross section, and results of numerical calculations are presented in five graphs. Multilegged cross sections are also considered and it is concluded that they possess more desirable characteristics as pressure responsive elements. An experimental study was made with two kinds of thin tubes, and qualitative agreement with theory was found.

W. Wuest, Germany

2736. Vreedenburgh, C. G. J., and Stokman, O., Some new elements in the calculation of flat slab floors, Anniv. Vol. appl. Mech., dedicated to C. B. Biezeno; Haarlem, Antwerpen, Djakarta, N. V. De Technische Uitgeverij H. Stam, 1953, pp. 253-281. Hfl. 20.

Paper discusses determination of bending moments in flat slab floors. Method consists in replacing supporting column by a fictitious clamping circle of radius R . The usual biharmonic equation is first solved for the slab, and the values of slope and radial moment evaluated round the origin (center of column) as a function of the clamping radius R . Both slope and moment vary almost linearly with R , and authors propose that this approximation be made. The column head is then analyzed, also in terms of a clamping radius, and the two solutions matched to give the combined action of slab and column. Use of R simplifies numerical work, and authors give detailed results for column heads of both hyperboloid and conical form. Results are well supported by tests on steel model. Authors consider advantages of fictitious clamping circle such that this should be basis of calculations and building code requirements for such slab floors.

J. Heyman, England

2737. Stippes, M., Large deflections of rectangular plates, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 339-345, 1952.

A procedure for the solution of the nonlinear fourth-order partial-differential equations known as von Kármán's equations for the large deflection of plates is given for a class of boundary conditions. This class of boundary conditions is characterized by the property that no forces exist in the plane of the plate at the boundary. Under these circumstances, a Ritz solution can be applied in a particularly simple fashion. The paper treats rectangular plates that are simply supported on two parallel edges, and either simply supported, free, or elastically supported on the remaining two parallel edges. A comparison of theory with experiment for a uniformly loaded square plate, simply supported on four sides, shows excellent agreement.

R. L. Bisplinghoff, USA

2738. Ramberg, W., and Miller, J. A., Determination of stress-strain curve in shear by twisting square plate, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 513-519, 1952.

Paper reports on a method for the determination of stress-strain curve in shear by twisting a square plate. Analytically, it is quite simple to show that if at a point the principal stresses are equal and opposite, then there exist planes on which there is pure shear.

In the present method, the equal and opposite principal stresses are produced by equal and opposite transverse forces applied to adjacent corners of a square plate. By means of equilibrium considerations, the maximum shearing stress can be expressed in terms of the loads.

By geometrical considerations, the maximum shearing strain can be expressed in terms of the deflection of the plate. The shear stress-strain relation can thus be determined from the load-deflection curve for the plate. This method is also extended to apply to stresses and strains in the plastic range.

Experimentally, it is rather difficult to satisfy all the assumptions made in the theoretical analysis. The authors realize this and discuss at length the inaccuracies due to edge effects, mem-

brane stresses, and deformation by transverse shearing force. As a check of their method, authors compared the octahedral stress derived from stress-strain curve in shear for a twisted plane with the octahedral stress derived from compressive and tensile tests of the same material.

Good agreement was obtained for an aluminum alloy but not for mild steel.
Y.-H. Pao, USA

2739. Noel, R. G., Elastic stability of simply supported flat rectangular plates under critical combinations of longitudinal bending, longitudinal compression, and lateral compression, *J. aera. Sci.* 19, 12, 829-831, Dec. 1952.

The critical stress coefficients for the buckling of simply supported flat rectangular plates under combined longitudinal bending, longitudinal compression, and lateral compression are found by the well-known energy method (Rayleigh-Ritz). Interaction curves for these loadings are presented for various plate-aspect ratios. Also included are curves of critical buckling constants for simply supported flat rectangular plates under combined unsymmetrical bending and lateral compression. These curves can be directly used to design a plate or to determine the stability of a given plate under such loading conditions. The loading condition assumed in the analysis is quite general and includes such cases treated by previous investigators as longitudinal bending and compression, longitudinal bending and lateral compression, and compression in two directions.

In following through the details of the paper, reviewer discovered more than fifteen misprints, most of which appear in the equations under "Theoretical analysis." Although they do not seem to have influenced the computations and resulting curves, readers are warned against using the equations in the paper. Reviewer hopes author will correct these misprints and confirm the validity of the curves in the Readers' Forum of the *Journal*.
G. C. K. Yeh, USA

2740. Galimov, K. Z., Equilibrium equations of the theory of elasticity for finite displacements and their application to the theory of shells (in Russian), *Izv. Kazan. Filial. Akad. Nauk SSSR Ser. Fiz.-Mat. Tekh. Nauk* 1, 25-46, 1948.

Author considers the geometry of finite strain and proceeds to express the equations of equilibrium in the form

$$A^{\alpha\beta}{}_{,\alpha} + \rho_0 f^{\beta} = 0$$

where $A^{\alpha\beta}$ is the contravariant component in the direction of the undeformed axis x^{β} of the stress vector acting on the deformed surface referred to unit area of the undeformed surface, which, as a result of deformation, goes into the deformed surface $x^{\alpha} = \text{const}$. This tensor is not symmetric but presents the advantage that the covariant derivation, indicated by the comma, depends only on the metric tensor of the undeformed state. The introduction of this tensor, in the theory of shells, leads to an additional term in the moment equation of equilibrium. He concludes that the momentless state in shells with finite displacements is practicable only when certain restrictive conditions are put on the displacements. The shell of variable thickness is next considered, and equations are derived which have already been given by Chien [*Quart. appl. Math.* 1, 297-327, 1944]. The method of derivation is, however, different.

L. M. Milne-Thomson, England

2741. Galimov, K. Z., The general theory of elastic shells with finite displacements (in Russian), *Izv. Kazan. Filial. Akad. Nauk SSSR Ser. Fiz.-Mat. Tekh. Nauk* 2, 3-38, 1950.

This is a continuation of the work of the paper reviewed above. The first section gives the theory of the deformation of surfaces in

general coordinates, and conditions of compatibility. The second gives the equilibrium equations of shells of undeformed thickness $2h$ referred to the metric of the undeformed shell. For this purpose the author introduces a moment tensor analogous to the stress tensor $A^{\alpha\beta}$ of the first paper, and the relation between the deformation and the external force system is examined. (Author claims to make no arbitrary hypotheses, but it seems to the reviewer that he tacitly assumes that the undeformed middle surface and its normals go over to the deformed middle surface and its normals.) The third section introduces boundary conditions in forms adapted to the deformed and undeformed states. In the fourth section the equations are formulated with the approximation of Kirchhoff's hypothesis. The fifth section gives the full set of equations of equilibrium and compatibility for thin shells. (This discussion appears to be not entirely satisfactory, for the author gives no explicit definition of "thin.")

L. M. Milne-Thomson, England

2742. Csonka, P., Contribution to the elasticity theory of circular-cylindrical shells (in German), *Acta Techn. Hung. Budapest* 6, 1/2, 167-174, 1953.

Author improves approximate solution for problem of bending of thin elastic circular cylinder by A. Aas Jacobsen [*Baugingenieur* 20, p. 394, 1939]. A method is presented for obtaining a solution of Flügge's differential equations of the shell loaded by tractive forces on its surface. Method of undetermined coefficients and use of a stress function enable author to achieve his results.

W. H. Hoppmann, II, USA

2743. Novák, O., Determination of stresses in some shells (in Czech), *Anniv. Volume Zdeněk Bažant, Praha, Technická Věda, Vydavat.*, 171-192, 1952.

In this study the author shows the determination of stresses in three types of shells without bending moment and submitted to vertical load uniformly distributed in projection. Approximation is done by the method called finite differences and by iteration; it gives the idea of stress functions, as well as of practical use of the determined stresses.

From author's English summary

2744. Tamate, O., Stresses in an infinite strip with a semi-circular notch under uniform tension and pure bending (in Japanese), *Trans. Soc. mech. Engrs. Japan* 18, 75, 7-15, Nov. 1952.

A plane stress problem about an infinite strip, which has a semicircular notch on one side of the straight edge, is solved. The method of solution is similar to that of Howland, namely, general formulas which give the coefficients in the required stress functions of each approximation are first derived linearly in terms of those of the preceding approximations, and the expressions of the stress components in the immediate neighborhood of the notch are formulated. The results of numerical calculations about the principal stress along the edge of the notch and the axial stress on the line of symmetry are described for the cases of a uniform tension and a pure bending, where the ratios of the radius of the notch to the width of the strip are 1/2, 1/5, and 1/3.

T. Udoguchi, Japan

2745. Eggert, T., Design of high-pressure pipe-line systems by means of shrinking (in German), *Maschinenb.-tech.* 1, 4, 5, 8; 177-184, 234-236, 363-369; July, Aug., Nov. 1952.

The purpose of this paper is the analysis of a composite cylinder made of two tubes assembled by shrinking. The stresses are investigated in the elastic domain as well as under the condition of partial plasticity in one of the two component tubes. The shrinking temperatures are calculated in detail in a series of numerical

examples. The questions of optimum proportions of the component tubes and of residual stresses are also discussed.

The treatment is conventional, and the analysis of plastic deformation follows the criterion of Hencky and von Mises; the difficulties arising from the discontinuity of Poisson's ratio at the boundary between elastic and plastic domain (see, e.g., R. Hill, "Plasticity") are not investigated. The axial elongation of the tube is supposed to be zero, in which case the use of the von Mises criterion is particularly simple.

This reviewer has found no essentially new contribution in this paper. The order of presentation followed by the author could be criticized. First, he gives the differential equation of equilibrium of an element and the classical expressions from Lamé

$$\sigma_r = A - B/r^2, \quad \sigma_\theta = A + B/r^2$$

without any demonstration (p. 177); then, at pages 182-84, he establishes the differential equation as well as the expressions for the strains ϵ_r and ϵ_θ in terms of the radial displacement u (expressions which, by the way, are not utilized at all in the paper). This strange treatment bears no comparison to the excellent exposition by S. Timoshenko, "Strength of materials," II, 236-244, pp. 389-395.

This reviewer is not sure that the problem investigated corresponds to the present state of the technique. It seems to him that the "autofrettage" process (which is not discussed at all in the paper) offers a better and cheaper means of obtaining the desired result.

C. Massonnet, Belgium

Buckling Problems

(See also Rev. 2772)

2746. Steele, T. K., and Wang, C. T., The effect of the torsional rigidity of a single stiffener on the buckling characteristics of a panel subjected to axial compression with large deflections, *Aero. Sci.*, 20, 1, 12-18, Jan. 1953.

Simply supported rectangular plate (width-length ratio 2:1) with single center stiffener is subjected to an end compressive load applied along the longer edges. Stiffener is assumed to have infinite bending rigidity, and (a) zero torsional rigidity or (b) infinite torsional rigidity. Hence, problem is reduced to compression of square plate with (a) all four edges simply supported, and (b) three edges simply supported and the fourth (unloaded) edge rigidly clamped. The von Kármán nonlinear equations are solved numerically by representing the deflection function, stress function, and fixed-edge moment function by infinite series. Comparisons are made between the critical buckling loads, effective plate widths, and unit plate shortenings for the two cases. Authors conclude that an outstanding advantage resulting from the use of a stiffener of high torsional rigidity is the substantial increase in the critical stress constant, a result that is fairly obvious, since they are effectively comparing a plate having all four edges simply supported with one of greater edge restraint.

Reviewer believes that there is no novelty in this paper, as it utilizes the identical procedure for solving the von Kármán equations which was worked out by Levy and referenced in the paper. The only difference is that a slightly different combination of boundary conditions is chosen. Reviewer also questions the validity of employing a divergent series to represent an auxiliary normal pressure distribution which replaces the fixed-edge moment distribution. This series was obtained from a Fourier series by a limiting procedure. If the Fourier series itself had been substituted into the lateral equilibrium equation, then the resulting series for the deflection function could perhaps have

been shown to be convergent by taking the limit. The paper contains several typographical errors.

H. Lurie, USA

2747. Chawla, J. P., Numerical analysis of the process of buckling of elastic and inelastic columns, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 435-441, 1952.

For an elastic slightly curved column, Hoff [AMR 4, Rev. 2434] presented an analysis of the process of buckling. In order to overcome the computational difficulties, this numerical method, which can be extended to study inelastic buckling, was developed. It is assumed that the velocity of the cross head of the testing machine is uniform, the column is of constant cross section, and the small initial curvature is of sinusoidal shape. Considerations of dynamic similarity lessen the work in the inelastic case.

Nondimensional charts are given which show deflection and load as functions of time and initial crookedness for elastic behavior. At first, dynamic deflections lag behind static deflections, then overshoot the static values and finally oscillate about the position of static equilibrium. The dynamic load can exceed the Euler load.

With inelastic behavior, the initial sinusoidal shape is not preserved. The polynomial expression used to describe the deflected shape needs further study before conclusions are drawn regarding buckling process.

Marshall Holt, USA

2748. Osgood, W. R., The effect of residual stresses on column strength, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 415-418, 1952.

A general expression is obtained for the buckling load of a column containing residual stresses assumed to be the same at every cross section and so distributed over the cross section that the Engesser-Shanley theory may be applied. An interesting example shows that the column curve of a rectangular column made of ideal elastic-plastic material with a parabolic distribution of residual stress is of the Gordon-Rankine type.

M-L. Pei, USA

2749. Gerald, G., Torsional instability of a long sandwich cylinder, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 391-394, 1952.

Paper is an analysis of torsional instability of a sandwich cylinder in which the core material is assumed to be isotropic. The bending rigidity of the cover sheets is assumed to be negligible. An assumed deflection pattern is used in conjunction with the modified Donnell equation of eighth order for a sandwich shell. Resulting substitution leads to a nontrivial solution for the critical torsional loading. This solution is minimized for the two cases: (a) core weak in shear, and (b) core not weak in shear. The assumed deflection pattern does not satisfy the boundary conditions, but for long cylinders this is unimportant. Author shows that even for the range of cylinder parameters encountered in design, the effect of length may be relatively unimportant. Results indicate that for case (a) the buckling load depends only on the shear rigidity of the core, while for case (b) the familiar two-lobe buckling for homogeneous cylinders occurs.

H. Lurie, USA

2750. Salvadori, M. G., Lateral buckling of beams of rectangular cross-section under bending and shear, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 403-405, 1952.

Values of bending moment which will cause lateral buckling are found for the case of a beam of thin rectangular cross section under a uniformly varying bending moment. The beam is con-

sidered to be simply supported in the lateral direction at the ends. A solution to the differential equation of equilibrium is obtained in terms of Bessel functions of order $\pm 1/4$ of the first kind. Numerical results are tabulated for values of the ratio of the bending moment at one end to the bending moment at the other and ranging from 1.0 to -1.0, at intervals of 0.1.

It is shown that a lower bound to the critical bending moment for a continuous beam may be found by calculating the critical loading for each span as though the span were simply supported in the lateral direction at the ends. This establishes a lower bound, since the span having the lowest critical load when simply supported in the lateral direction will be restrained somewhat by adjacent spans.

J. W. Clark, USA

2751. Wittrick, W. H., Lateral instability of rectangular beams of strain-hardening material under uniform bending, *J. aero. Sci.* 19, 12, 835-843, Dec. 1952.

Recently, Neal discussed the lateral buckling of a thin, deep, rectangular cross-sectioned beam under uniform bending and such that the stresses were sufficiently large to cause plastic flow to occur. His work also included experiments on beams of annealed mild steel.

Present paper discusses a generalization of the same problem. By analogy with recent column treatments, two extreme cases are considered for strain-hardening materials, namely, (a) "lowest critical bending moment at which lateral deflection and twist can occur with increasing bending moment," and (b) "upper critical bending moment at which, if no previous lateral deflection has occurred, the beam will buckle laterally under constant bending moment."

After general treatment, numerical calculations are made for an aluminum alloy and for annealed mild steel. Author found that, for Neal's material, the two critical values are substantially the same; for the aluminum alloy, however, they vary by about five per cent.

F. S. Shaw, USA

2752. Hill, H. N., and Clark, J. W., Lateral buckling of eccentrically loaded I- and H-section columns, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; *J. W. Edwards, Ann Arbor, Mich.*, 407-413, 1952.

Article describes tests and analysis of lateral buckling of three different cross sections of aluminum-alloy columns subjected to simultaneous end thrust and bending in the plane of the web. Members were tested with equal eccentricities at both ends. Testing heads provided essentially a hinged condition for bending in the direction parallel to the web, and a flat end condition for bending normal to the web. Failures were obtained in both the elastic and plastic stress regions.

Authors find interaction-type solution which gives close agreement with test results. For this comparison the tangent modulus theory is used to determine critical values of thrust and end moment in the plastic region. A simplified interaction formula (similar to the usual straight line equation but with a correction to take account of the increase in moment due to the deflection of the members) is recommended for the design of members loaded in compression and bending, which fail by lateral buckling.

N. M. Newmark, USA

2753. Melcon, M. A., and Ensrud, A. F., Analysis of stiffened curved panels under shear and compression, *J. aero. Sci.* 20, 2, 111-119, 126, Feb. 1953.

New formulas are derived for the critical buckling stress of curved panels in shear and for the effects of the diagonal tension field during the postbuckling state. Latter effects are expressed by fictitious compressive stresses in the stiffeners, which are com-

bined by an interaction equation with the compressive stresses due to external loading. Formula for the ultimate shear strength of sheet is given. Methods of paper agree well with experiment and are readily adaptable to high-speed computing techniques. Sample interaction diagram shows how results may be presented for practical application.

From authors' summary by H. D. Conway, USA

2754. Federhofer, K., Stability of a circular cylindrical shell with variable wall thickness (in German), *Öst. Ing.-Arch.* 6, 4, 277-288, Sept. 1952.

Author discusses the stability of a circular cylindrical shell of continuously variable wall thickness, loaded by axial pressure. Under the assumption that buckling is axially symmetrical and by choosing the variables in a suitable manner, the fourth-order differential equation which governs the problem is put into a simple form. Resolution of this equation by known functions being impossible, the critical load is computed (under the assumption of a slightly varying wall thickness) by using the approximation theory of eigenvalue problems. The method is applied to linearly and quadratically varying wall thicknesses.

From author's summary by C. B. Biezeno, Holland

2755. Lin, T. H., Shortening of column with initial curvature and eccentricity and its influence on the stress distribution in indeterminate structures, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; *J. W. Edwards, Ann Arbor, Mich.*, 449-452, 1952.

Paper gives method of calculating shortening of compression members by assuming initial deflections as Fourier series. Two examples are worked of indeterminate structures, which show considerable (15%) difference in stresses according to whether shortening is allowed for or neglected.

J. Heyman, England

Joints and Joining Methods

2756. Dresden, D., Shrink-fit used to transmit a torque, *Anniv. Vol. appl. Mech.*, dedicated to C. B. Biezeno; Haarlem, Antwerpen, Djakarta, N. V. De Technische Uitgeverij H. Stam, 1953, pp. 225-230. Hfl. 20.

The theory of partially plastic thick cylinders is used to estimate the torque capacity of a heavy shrink fit. In the plastic region the material is assumed to yield at constant shear stress. Equations developed may be found in standard books on advanced strength of materials.

W. O. Richmond, Canada

2757. Spies, G. J., The peeling test on redux-bonded joints, *Aircr. Engng.* 25, 289, 64-70, Mar. 1953.

A theoretical analysis is given of the peeling test for Redux-bonded joints as devised by Aero Research, Ltd., and generally accepted as a standard quality control test for metal-bonding processes.

Numerical values derived from computations appear to be in reasonable agreement with experiments, but more test data and better knowledge of the elastic and plastic behavior of both adhesive and adherent are necessary to make the method more reliable in this respect. The practical value of the method, however, is the indication it gives about the sensitivity of the test for the variables involved. This may make it feasible to introduce corrections for variations of some parameters, thereby improving the reproducibility of the test.

From author's summary by Wm. J. Carter, USA

Structures

(See also Revs. 2755, 2778, 2785, 2798, 2800)

2758. Roccatelli, C., *Elements of civil engineering*. Vol. 1. *Masonry structures* [Elementi delle costruzioni civili. Vol. 1. *Strutture murarie*], Pisa, Colombo Cursi, 1950, xi + 395 pp. Lire 6700 (vols. 1, 2).

This textbook is, in many aspects, different from similar books presenting the subject. Its fundamental aspect is to present the elements of civil engineering not only from static standpoint but also in connection with their formal characteristics and aesthetic analysis. Especially the intuition of the student is emphasized, so important for the structural sense. Problems are discussed which very often are overlooked in similar textbooks, as, e.g., problems of soil mechanics in connection with design of foundations, methods of mechanical and chemical consolidation of bearing soils, underground masonry, scaffolds, etc. All valuable types and methods of construction in the past are reviewed and compared with recent technical progress, with special consideration of economy. Illustrations are very clear and organic, all of them prepared for this book which, in its two volumes, is not only an excellent guide for students of engineering but also an exhaustive handbook for practicing civil engineers. Vol. I includes: Foundations, consolidation of foundation soils, distribution of soil pressure under various types of foundations, slides, excavation, masonry buildings, structural design of walls, scaffolds, bracing, vaults, floors, stairways, roofs, roofings.

J. J. Polivka, USA

2759. Roccatelli, C., *Elements of civil engineering*. Vol. 2. *Reinforced concrete and steel structures* [Elementi delle costruzioni civili. Vol. 2. *Strutture in cemento armato e in acciaio*], Pisa, Colombo Cursi, 1950, viii + 368 pp. Lire 6700 (vols. 1, 2).

The second volume of this outstanding textbook deals with structures in reinforced concrete and in steel, and presents them in the same original way as masonry structures in vol. 1. This is a field in which many new structural types were created recently (long-span and space structures). Following subjects are treated and illustrated, with references to the latest structural achievements: (A) Reinforced concrete: fundamentals of reinforced concrete; forms and scaffolds; structural principles in general; structural elements in reinforced concrete; precast (built-up) structures; underground structures; columns and piers; beams in tension; horizontal structures (floors with various elements, conditions of stability); stairs; roofs; cantilever structures; shell-type structures and domes; trusses; hollow floors. (B) Steel: materials and shapes; methods of connections; general layout of steel structures; foundations; riveted and welded structures; lattice work; trusses and open-web girders; columns and towers; frame structures, stairs; floors; roofs; fireproofing and veneer.

J. J. Polivka, USA

2760. Kooharian, A., *Limit analysis of voussoir (segmental) and concrete arches*, *J. Amer. Concr. Inst.* 24, 4, 317-328, Dec. 1952.

This is a stimulating paper dealing with plastic analysis of arches having no tensile strength. Problem is treated under assumption that, in the plastic hinges, thrust-line passes through edge of voussoirs, thus implying infinite compressive strength. Internal work done in hinges then is zero, which simplifies procedure when Greenberg and Prager's upper and lower bound theorem [AMR 3, Rev. 2273] is applied. Among other problems, the load is determined which will cause an arch of known weight and dimensions to collapse if acting at a given point. No clear

proof, however, is given for correct positioning of plastic hinges in that case. Topic of paper is also dealt with in a different way and more elaborately by reviewer [AMR 6, Rev. 1566] who also introduces effect of noninfinite compressive strength.

H. Craemer, Germany-Egypt

2761. Tesař, V., *Contribution to the solution of three-dimensional framed systems* (in Czech), *Anniv. Vol. Zdeněk Běžant*, Praha, Technic.-Vědec. Vydavat., 193-212, 1952.

Paper describes the method of solving axial forces of a two-joint bridge arch (span 345 m) unsymmetrically loaded by side wind. The arch has the form of a spatial, four-lateral structure of which two opposite walls are plane. Effect of the unsymmetrical side wind is considered as a loading antisymmetrical to the longitudinal axis of symmetry of the arching. Equilibrium of external and internal forces at particular points of contact of the bars leads to recurrent formulas for axial forces in transverse diagonals. Successive elimination then gives final expressions for axial forces. There is also mentioned a special case, when the side-wind loading is symmetrical according to the transverse plane of symmetry of the arch.

Method is also applicable for solving spatial structures having one axis of symmetry even under generalized loadings, which must then be expressed by superposition of their symmetrical and antisymmetrical components according to the longitudinal plane of symmetry.

V. Kopriva, Czechoslovakia

2762. Halbritter, F., *Analysis of spiral stairs* (in German), *Beton u. Stahlbeton*, 47, 7, 159-163, July 1952.

Stair is idealized into a horizontal plate having the shape of a ring sector freely supported on outer contour and two radii under equally distributed load. Polar coordinates are used, and known solution by extension of load into trigonometrical series of angular coordinate is applied. It is found that main values for moments, etc., depend rather linearly on magnitude of angle of ring sector. This is used to derive several simple approximative formulas. An illustrative example is added.

H. Craemer, Germany-Egypt

2763. Löfquist, B., *Calculating a concrete core wall*, *Trans. Fourth Congr. inter. Comm. Large Dams*, New Delhi, Jan. 1951, in 4 vols., vol. I, pp. 111-133. \$48 per set.

Methods of calculating stresses and deflections are given that check closely with measured deflections of core walls in Swedish dams. Stresses examined are due to: (1) Primary bending due to horizontal water load for various connections of wall to foundation; (2) secondary bending due to irregularities in fill and its settlement; (3) vertical compression due to friction of fill during settlement; and (4) shrinkage and temperature. A thin core wall is advantageous with widely distributed irregularities in fill, whereas a thick wall possesses greater capacity for withstanding more concentrated irregularities. The vertical pressure is decisive as to thickness in large dams. A simple method is given for calculating the reinforcement necessary to prevent the formation of large individual cracks. This paper is a significant contribution to an important and difficult problem.

J. C. Geyer, USA

2764. Mazure, J. P., *Statical problems in the code of practice for steel windows*, *Anniv. Vol. appl. Mech.*, dedicated to C. B. Biezeno; Haarlem, Antwerpen, Djakarta, N. V. De Technische Uitgeverij H. Stam, 1953, pp. 283-297. Hfl. 20.

Author analyzes metal window sash (with glass in place) in order to obtain a rational basis for designing or for speci-

lying details of windows. No attempt at a complete design is made, i.e., starting with known forces and obtaining definite dimensions. (As author points out, the loads on window sash cannot be specified with any certainty.) Instead, author proposes to use analysis to extrapolate from existing designs, known to be satisfactory, to other shapes and sizes. Analysis indicates the manner in which dimensions of metal sections and thickness of glass will vary with size and shape of window.

M. P. White, USA

2765. Brock, J. E., A matrix method for flexibility analysis of piping systems, *J. appl. Mech.* 19, 4, 501-516, Dec. 1952.

By use of matrix algebra, a concise and strongly motivated method is developed for analyzing flexibility of piping systems. The method is particularly suited for systems which are complicated by multiple branches, additional constraints, inconvenient orientations, and so on, as well as for examination and modification for particular purposes of more widely known methods.

From author's summary by H. Becker, USA

2766. Sandwich construction and core materials. Part VI. Section I. Pullen, W. J., "Balsolite" impregnated paper cellular material as an elastic stabiliser; Section II. Chapman, R. G., and Pearson, S., Compression tests on sandwich panels with "Balsolite" cores; Section III. Oaks, J. K., Strength tests of a typhoon type fuselage of "Balsolite" sandwich construction *Aero. Res. Coun. Lond. Rep. Mem.* 2687, 18 pp., Feb. 1948, published 1952.

Compression tests on struts, panels, and static bending tests of a typical fuselage construction using sandwich construction with Balsolite core material are reported. In section I, additional check points are provided for the well-known relation between the classical Euler load and the shear stiffness of the strut. It is also concluded that struts with Balsolite core compare favorably with other constructions using low-density core materials. In section II, it is concluded that panels using birch plywood facings have higher panel efficiencies than panels using paper laminate facings. In section III, a typhoon-type fuselage of pure monocoque construction in the shape of a frustrum of a cone of elliptical section tested in both vertical and lateral bending exhibits very little skin wrinkling right up to the failing load. The relations between deflection, strain, and the applied loads appear to be linear up to the failing load. C. C. Wan, USA

2767. Structural principles and data, Published under the authority of the Council of The Royal Aeronautical Society; New York, London, Toronto, Pitman Publ. Corp., 1952, xii + 322 pp. \$8.50.

Book is divided into two parts. Part 1 (73 pages), Structural airworthiness, covers "(a) the selection of maneuvering and other conditions appropriate to the intended use of the aeroplane, and the specification of these in terms of design cases; (b) the estimation of the loading systems acting on the aeroplane structure arising from these design cases." Part 2 (244 pages), Structural analysis, covers "the estimation of the stresses and hence the over-all strength of the aeroplane structure under the applied loading system."

Topics discussed in part 1, including formulas, are symmetric maneuvering loads, gust loads, tail loads, aileron loads, landing gear loads, engine-mounting loads, crash loads, factors of safety, repeated loads, and fatigue based on "British Civil Airworthiness Requirements," "U.S. Civil Air Regulations, 3, 4b," and "Design requirements for aeroplanes for the Royal Air Force and Royal Navy."

Part 2 covers in considerable detail with discussion, formulas, tables, and charts such topics as stress-strain relations; properties of aluminum alloys; beams and struts, including elementary bending and torsion, torsion-bending and torsional instability of thin-walled open sections, "coupled flexural-cum-torsional instability," and local instability of struts; instability and postbuckling behavior of plates and panels in compression; optimum weight design of compression panels; buckling and postbuckling behavior of cylinders and curved plates in compression; buckling under shear and combined loadings; flat and curved shear panels in incomplete diagonal tension field as based on NACA reports by Kuhn and others; wing analysis for open and closed box beams and multicell box beams; shear-lag effects, including an approximate solution at wing cut-outs; rings and curved beams; effect of flexibility on fuselage rings. Many of the tables and diagrams are taken from Roark, "Formulas for stress and strain," and "Stressed skin structures data sheets" of the Royal Aeronautical Society. Over 300 items are listed in the references and bibliography.

The book should be a useful addition to the aircraft structures engineer's library and should prove useful as a reference for students studying aircraft structures. B. E. Gatewood, USA

2768. Benthem, J. P., On the stress analysis of swept wings (in Dutch), *Nat. LuchtLab. Amsterdam Rep.* S.405, 103 pp., 61 figs., Sept. 1952.

A good deal of ground has been covered in this report. A detailed discussion of the recent literature on sweptback wings is given and a comprehensive bibliography included. The use of oblique coordinates in the investigation of oblique panels under shear and direct stresses is interesting, though not entirely new. The reviewer feels that the report is presented well, is readable and gives a good review of the possible scope of future development and the work done hitherto by other workers in the field.

Y. V. G. Acharya, Holland

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 2705, 2730, 2745, 2949)

2769. Hetényi, M., A study in photoplasticity, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 499-502, 1952.

This exploratory investigation of a specially prepared nylon copolymer is to determine its suitability for photoplasticity studies. It was found that, in addition to exhibiting appreciable birefringence under load, this material has a well-defined yield point and can undergo large deformations at nearly constant stress, thus approximating the behavior of low-carbon steels. Although the stress-strain and strain-optical characteristics are affected by relative humidity, the most striking property of this material is the completely linear strain-optical relation which extended far beyond the yield point. In addition to the physical properties, fringe pattern photographs of various slip-band formations are given. G. Gerard, USA

2770. Edelman, F., On the coincidence of plasticity solutions obtained with incremental and deformation theories, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 493-498, 1952.

Ilyushin's power law [AMR 2, Rev. 731] for strain in work-hardened material is examined and shown to be applicable to a great variety of loading criteria. It is also shown that, under normal conditions, this power law is both necessary and sufficient.

But, as is seen by the work of Prager and Winzer [AMR 1, Rev. 162], such a law cannot be accepted in the case of small strains. The same conclusion can be arrived at from Fig. 1 of the present paper. Even in the case of large strain the agreement with experimental results is rather poor. The reviewer feels that, in the case of strains well beyond the elastic limit, Blyshin's hypothesis can be accepted, with caution, until there is more evidence in support of or in contradiction to it.

Y. V. G. Acharya, Holland

2771. Findley, W. N., Derivation of a stress-strain equation from creep data for plastics, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 595-602, 1952.

Some of the conditions under which a mechanical equation of state may exist have been reviewed and tested in particular for a canvas laminated plastic. In addition, an equation has been developed which represents the creep behavior of canvas laminate and is in agreement with pertinent theory. From this equation for creep vs. time at different stresses, an expression has been derived for stress vs. strain in a tension test at known rates of strain. The agreement between the predictions of this equation and the tension test data at different strain-rate histories lends support to the existence of a mechanical equation of state for this material. However, there are indications that at best the mechanical equation of state may be only an approximation.

From author's summary by E. A. Davis, USA

2772. Rosenthal, D., and Baer, H. W., An elementary theory of creep buckling of columns, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 603-611, 1952.

An elementary theory of creep buckling of columns with small initial eccentricity has been developed on the basis of a series of simplifying assumptions. It was assumed that the deformation tended to a finite value under a constant stress in the case of the transient creep and that it progressed at a constant rate in the case of the steady-state creep. Under these circumstances, the theory predicted the existence of a criterion of creep buckling for an indefinite time of service if the creep in tension was transient. This criterion was equivalent to the Euler formula for buckling (as modified by the use of the tangent modulus). If the creep contained a steady-state component, the theory showed that the column would become rapidly unstable under any stress.

The existence of a critical stress in creep buckling has been demonstrated in a series of preliminary tests of short duration (48 hr) on 99.5% aluminum at 95 F.

From authors' summary by J. W. Clark, USA

2773. Rao, K. L., A rational conception of creep in concrete, *Trans. Fourth Congr. Inter. Comm. Large Dams*, New Delhi, Jan. 1951 (in 4 vols), vol. III, 307-333. \$48 per set.

Author maintains that what is called "creep" in concrete (i.e., either a slow viscous flow or a delayed elastic strain, both roughly proportional to the stress) is actually mainly, if not totally, due to shrinkage through loss of moisture, the movement being considerably reduced when the specimen is coated with asphalt or kept wet. In a beam reinforced on the tension side, the reinforcement opposes the shrinkage of adjacent material. The shrinkage across the section is, therefore, not uniform, but varies according to the position of the reinforcement and also the amount of freedom of movement of bottom of beam, being greatest on the top. This eccentricity of deformation produces additional deflections increasing in time, thus simulating "rheological" creep. It is stated that the "average" shrinkage of top and bottom is

the same for both loaded and unloaded beams and that there is no appreciable increase of steel stress due to creep, the movements being everywhere practically independent of the stress. While this theory is supported by experimental results and reference to Slater and Inge Lyse [*J. Amer. Concr. Inst.*, 1950], it is contrary to classical careful observations of Glanville and Thomas [*Building Research technical Papers*, 21, 1939] in which shrinkage was separated from creep as shown in Fig. XIII.5 of M. Reiner, "Deformation and flow—an introduction to theoretical rheology" [AMR 4, Rev. 1159]. Because of startling results, paper is recommended to workers in this field for critical study.

M. Reiner, Israel

2774. Stroh, A. N., The mean shear stress in an array of dislocations and latent hardening, *Proc. phys. Soc. Lond. (B)* 65, part 1, 397B, 2-6, Jan. 1953.

The root-mean-square shear stress on a slip plane due to an array of dislocations is taken as a measure of the work-hardening of that plane. The ratio of the hardening of the latent slip system to that of the operative system, which is shown to be independent of the distribution of the dislocations, is calculated both for screw and for edge dislocations. Numerical values are given for the twelve slip systems of face-centered cubic crystals. The results suggest that hardening by edges may account for the greater part of the observed latent hardening.

From author's summary by John C. Fisher, USA

2775. Inoue, N., Application of gas dynamical method to soil mechanics and theory of plasticity. I, II, *J. phys. Soc. Japan* 7, 6, 604-618, Nov.-Dec. 1952.

It is shown that the mathematical theories of plastic deformation or seepage of granular material are isomorphous to the theory of irrotational supersonic flow of gases in two dimensions by substituting the flow vector of the gas for a simple function of the stresses. This isomorphism is used for solving problems in plasticity in terms of aerodynamical concepts. Author considers the straining of a plastic body due to a penetrating cylindrical piston and a number of other examples, part of which admit a very favorable comparison with known experimental facts. It should be noted, however, that the pressure-density relations as assigned to the gases are arbitrary to such an extent that it does not seem possible to convert actual aerodynamical observations to statements on plasticity; if, on the other hand, the isomorphism is regarded as a new mathematical technique, a discussion of its merits relative to older methods would be desirable.

R. Eisenschitz, England

2776. Craemer, H., A theory of plastic bending from a statistical aspect, *J. appl. Mech.* 19, 3, 247-249, Sept. 1952.

The development of yielding during a bending test is a controversial subject. According to the classical viewpoint, the external fibers yield first at the tensile yield stress for the material, and yield gradually penetrates the cross section. More recently, the view has been advanced that the stress distribution remains linear until a stress is reached which is much higher than the yield stress from the tensile test, following which almost the whole cross section rather suddenly becomes plastic.

The title paper discusses this phenomenon from the statistical point of view. The cross section is assumed to be composed of a large number of elements of varying strength. The bending yield is determined by the weakest element near an extreme fiber; this element is usually stronger than the weakest element in the entire cross section, which determines tensile yield. This qualitative explanation is made quantitative by an analysis based on the distribution proposed by Weibull for strength prob-

lems. For a bar of rectangular cross section, for example, the ratio of tensile to bending yield is found to be about 1.5 if the ratio of standard deviation to average yield of the tensile elements is about 0.3. The relative yielding of weakest elements in tension and bending thus has a simple explanation; however, reviewer is unable to see why plastic action should be determined by the weakest element rather than by averages.

S. B. Batdorf, USA

2777. Gaydon, F. A., An analysis of the plastic bending of a thin strip in its plane, *J. Mech. Phys. Solids* 1, 2, 103-112, Jan. 1953.

Only bending by end couples sufficiently large to cause the whole volume to become plastic is considered. Problem is considered as one of generalized plane stress. Incompressibility and no strain hardening are assumed, and the Levy-von Mises flow theory is used. Solutions with both the maximum shear theory and an approximation to the distortion energy yield criterion are obtained. Full account is taken of the variation in thickness and the movement of the neutral surface. Results differ in some respects from those of Lubahn and Sachs [AMR 4, Rev. 132]. Reasonable agreement with experimental work by Sangdahl, Aul, and Sachs [AMR 2, Rev. 56] is shown for external circumferential strains.

A. D. Topping, USA

2778. Koiter, W. T., On partially plastic thick-walled tubes, *Anniv. Vol. appl. Mech.*, dedicated to C. B. Biezeno: Haarlem, Antwerpen, Djakarta, N. V. De Technische Uitgeverij H. Stam, 1953, 233-251. Hfl. 20.

Problem of thick tubes under internal pressure is attacked, using maximum yield criterion and associated incremental stress-strain relations, i.e., of the Saint Venant type, which in this case may be expected to give nearly as good approximation as those of the Prandtl-Reuss type, associated with the von Mises yield criterion. Solution is given in closed form for the case of plane strain as well as for the general case for tubes with closed or open ends. Applicability of solutions requires certain limitations for the ratio of external to internal diameter. Modifications for very thick tubes are discussed. Comparison with Hodge and White's much more complicated, rigorous solution, based upon the von Mises criterion [see AMR 4, Rev. 1568], shows deviations less than 25% for axial stress.

F. K. G. Odqvist, Sweden

2779. Woodcock, F. J., and Weiss, K. R., A method of measuring post-yield strain, *J. roy. aero. Soc.* 57, 505, 49-51, Jan. 1953.

The development of a directly bonded resistance type of strain gage for measurement of transient strains of the order of 5% is presented. The gage developed was 56% copper, 44% nickel, 0.0032 in. in diam, double cotton-covered Eureka wire. Durofix adhesive was used directly with the wire. Test results presented show breaking strains for this gage to be greater than 8% in all cases, and in one case greater than 15%. The significant characteristics are also discussed.

C. O. Dohrenwend, USA

2780. Urie, V. M., and Wain, H. L., Plastic deformation of coarse grained aluminum, *J. Inst. Metals* 81, 153-159, 1952/1953.

Authors reproduced fine grid photographically on specimen surface and measured local elongation over 0.5-mm gage lengths along traverses parallel to direction of applied tensile load. Data indicate that elongation varied widely from grain to grain and within individual grains of aggregate. Elongation was generally restricted in vicinity of grain boundaries, and form of restriction

appeared to depend on orientation relationship between grains. Fluctuations in local elongation, which accompanied occurrence of deformation bands, became larger as over-all elongation was increased.

G. M. Sinclair, USA

Failure, Mechanics of Solid State

(See also Revs. 2712, 2774, 2779, 2793, 2795, 2808)

2781. McLean, D., Crystal slip in aluminium during creep, *J. Inst. Metals* 81, 133-144, 1952/1953.

Paper deals with problem of clearly elucidating the nature of slip in metals, and reports details of slip appearance during creep of pure aluminum obtained from a variety of microscopic techniques other than electron microscopy. Two types of slip were observed—prominent slip bands $\frac{3}{4}\mu$ displacement and 30- μ spacing, and fine slip bands of 50-500 Å displacement and less than 1 μ apart. At all stages of creep a larger proportion of the flow occurred by fine slip. Greater proportions of plastic flow occurred by prominent slip with an increase in stress or grain size. Prominent slip differed from fine slip in degree rather than kind, since it was found that prominent slip was a localized concentration of fine slip. Dislocation mechanisms for producing the fine and prominent slip were described.

A. N. Holden, USA

2782. Bullen, F. P., Head, A. K., and Wood, W. A., Structural changes during the fatigue of metals, *Proc. roy. Soc. Lond. (A)* 216, 332-343, Feb. 1953.

X-ray diffraction and metallographic examinations of annealed copper subjected to cyclic stress showed two types of deformation, depending on the frequency of cycling. Slow (200 cpm) cycling caused extensive crystal breakdown similar to that observed with static stress, and the microscopically observed deformation zones were spread throughout the grains. When the same stress cycle was built up at a faster rate (1800 cpm), there was no x-ray evidence of disorientation, and deformation bands were localized. Movements in these bands do not lead to strain-hardening, but finally result in formation of a fatigue crack.

J. A. Bennett, USA

2783. Head, A. K., The mechanism of fatigue of metals, *J. Mech. Phys. Solids* 1, 2, 134-141, Jan. 1953.

Review of the work of several investigations shows that a fatigue failure can be considered to take place in three stages. In the first—which may be absent for high strength materials—bulk plastic deformation and work-hardening to the level of the applied stress occur. During the long second stage a crack is usually assumed to start in a region of localized deformation; the possibility exists that the deformation is localized to regions of already existing submicroscopic cracks. The third stage is considered to start when the crack has grown to visible size; the crack extends and sudden failure follows when the cross-sectional area is sufficiently reduced. No new theory is put forward to explain the mechanism of crack formation or propagation in terms of atomic behavior.

G. Sved, Australia

2784. Aronofsky, J., The formation of a necked region and fracture along an oblique line in flat tensile bars, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 521-532, 1952.

Following a discussion of earlier work on plastic flow in tensile tests of flat bars, particularly as regards oblique necking and failure, several series of tests of flat bars are described. In a series of tests of low-carbon steel bars with variable width-to-thickness ratio b/h it was found that, for small b/h , a symmetri-

cal neck was formed by contraction of the width, while for b/h greater than 8 an oblique neck was formed with very little contraction along the necked portion. For a series of strain-hardened steel bars, the necking was found to change from symmetrical to oblique as the strain-hardening was increased. In a bar with photogrid lines in the direction and transverse to the direction of pull, it was found that even after oblique fracture the lines remained orthogonal. The author shows that the flow theory, in which the octahedral shear stress is held constant, may be used to predict the formation of the oblique neck in a strain-hardened tension strip.

S. Levy, USA

2785. Walker, P. B., Design criterion for fatigue of wings, *J. roy. aero. Soc.* 57, 505, 12-18, Jan. 1953.

Early unsatisfactory criterion for required alternating load range in aircraft wing structures, $\pm 7\frac{1}{2}\%$ of static ultimate load, and 2×10^6 repetitions, has gradually been adapted to give a more reasonable criterion.

R. N. Arnold, Scotland

2786. McPherson, A. E., A dual-amplitude axial-load fatigue machine, *Proc. Soc. exp. Stress Anal.* 10, 2, 143-152, 1952.

The fatigue machine described herein is capable of applying axial loads that are (1) of constant range, (2) of different ratios of cycles at one stress to cycles of the other, or (3) of either the high- or low-stress amplitude first.

The machine was developed for testing thin gage aluminum-alloy specimens. While the main test program is to follow, preliminary tests indicate that results can generally be duplicated within less than 10%. Such agreement is quite satisfactory.

G. N. Cox, USA

2787. Foulkes, J., Minimum weight design and the theory of plastic collapse, *Quart. appl. Math.* 10, 4, 347-358, Jan. 1953.

This paper examines the problem of assigning economical sections to the members of a structure whose geometrical form is given. The criterion of failure is taken to be that of the plastic theory of collapse, and the criterion of minimum weight is employed to determine the best design. A geometrical analog of the equations involved is used to clarify their significance, and such proofs as there are in the text are cast into geometrical terms. A method of solution is suggested at the end of the paper, but the primary concerns of the paper are the general features of the problem.

From author's summary by G. Gerard, USA

2788. Cliett, C. B., Flexural fatigue strength of anodized 24S-T aluminum-alloy sheet, *Aero. Engng. Rev.* 11, 12, 29-30, 42, Dec. 1952.

The results of an investigation to determine the effect of anodizing on the flexural fatigue strength of 24S-T aluminum-alloy sheet are presented. The conclusion drawn is that anodizing reduces the life of the aluminum alloy when subjected to fatigue conditions and when subjected to the combined action of corrosion and fatigue.

From author's summary

2789. Hardrath, H. F., and Utley, E. C., Jr., An experimental investigation of the behavior of 24S-T4 aluminum alloy subjected to repeated stresses of constant and varying amplitudes, *NACA TN* 2798, 23 pp., Oct. 1952.

A device for adapting R. R. Moore rotating-beam fatigue-testing machines for tests in which the amplitude of stress is continuously varied is described. Tests of 24S-T4 aluminum-alloy specimens subjected to stresses of constant amplitude and to stresses with amplitudes varying according to sinusoidal and ex-

ponential functions are reported. The results are analyzed by computing the summation of cycle ratios. The values obtained in this analysis were found to be influenced by the shape of the frequency-distribution curve.

From authors' summary

2790. Terao, N., Statistical analysis of the resistance of glass to fracture—on the influence of temperature (in French), *J. phys. Soc. Japan* 7, 6, 627-630, Nov.-Dec. 1952.

Paper is concerned with the probability of fracture when a beam of glass is subjected to a uniformly increasing load. The application of Boltzmann statistics together with physical assumptions regarding the activation energy of the process leads to a relation between probability of fracture and temperature. Experimental work was carried out at a rate of loading of 8.56 gr/sec and at six different temperatures from 10 C to 300 C inclusive, for a number of samples ranging from 50 to 100 at each temperature.

A similar treatment was carried out by the reviewer [see *J. Chem. Phys.* 15, 10, 760-761, Oct. 1947].

E. Saibel, USA

2791. Durelli, A. J., and Okubo, S., Influence of strain gradient on brittle coating sensitivity, *Prod. Engng.* 24, 1, 136-137, Jan. 1953.

Authors use a common calibrating beam, loaded as a cantilever by an eccentric wheel, in which three additional holes are machined. These holes are placed so as to give deflections of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$ of the full 0.592-in. deflection, or the respective fractional parts of a 0.0002 (in./in.)/in. strain gradient. Magnaflux Corporation's Coatings nos. 1204-1207 are used. Curing temperatures are chosen to provide strain sensitivities from 0.0009 to 0.00035 in./in. at full deflection. Results of tests are shown by a family of curves. Curves show that the influence of strain gradient is applicable for the low strain-sensitivity coatings and gradually decreases as the strain sensitivity increases. (Coating of strain sensitivity 0.00035 in./in. at full deflection shows 31% less strain sensitivity at $\frac{1}{4}$ of full deflection.) An approximate formula gives curves which agree well with experimental curves. As an example, authors show how correction of results from brittle-coating stress analysis of a steel valve body reduces the stress at a special point about 16%. In an earlier paper [SESA Fall Meeting, 1950], authors studied the influence of other variables on the accuracy of coating determinations. [See also *AMR* 4, Revs. 1962, 2395; 5, Rev. 1975.]

E. Stenroth, Sweden

Material Test Techniques

(See also Revs. 2725, 2757, 2766, 2780)

2792. Chagneau, A., The impact bend test (in French), *Ann. Inst. tech. Bât. Trav. publics* 6, 62, 157-168, Feb. 1953.

The effect of the large number of variables in the notched-bar impact test is discussed. Included are those connected with the size and form of the test piece and those determined by the conditions of the test, such as striking velocity and temperature. A section is also devoted to the metallurgical factors involved, such as aging, structure, and heat treatment, and to the influence of the important alloying elements and impurities in steel.

It is considered that a single test at room temperature is not sufficient to test for brittleness. The results of a series of tests on Armco iron carried out at the Laboratoires du Bâtiment et des Travaux Publics with several values of notch radius and over a range of temperature serve to reinforce this conclusion.

A. F. C. Brown, England

2793. Boyd, G. M., The assessment of notch ductility by a variety of notch tests, Symp., Notch bar testing and its relation to welded constr., London, Inst. Weld., 11-18, 1953.

A wide variety of notch tests and impact tests of arbitrary nature are currently used to measure "toughness" qualities of steels for various purposes. This paper describes eight different test procedures, presents data from various sources in the literature, and, where possible, compares results obtained, such as transition temperatures. Author makes critical comments, and concludes that no clear relationship can be established between the indications of different types of notch test, even when the tests are performed on the same material.

P. E. Sandorff, USA

2794. Barr, W., Notch bar testing and the selection of steel for welded construction, Symp., Notch bar testing and its relation to welded constr., London, Inst. Weld., 7-10, 1953.

A general discussion of laboratory tests and their interpretation is reviewed from the viewpoint of practical considerations. Emphasis is placed on means by which the engineer or metallurgist may gain some indication of the probable service behavior of different steels. Adaptability to acceptance testing is discussed.

T. J. Dolan, USA

2795. Chalmers, B., The preparation of single crystals and bicrystals by the controlled solidification of molten metals, *Canad. J. Phys.*, 31, 1, 132-146, Jan. 1953.

Studies of fundamental properties of metals have been accelerated by use of single crystal and bicrystal specimens. Paper describes techniques developed for production of single crystals and bicrystals of a wide variety of metals by controlled freezing. Control of orientation by means of "seeding" with a previously prepared crystal of known orientation is discussed. Author also gives detailed analysis of factors influencing nucleation and growth of "stray" crystals which interfere with desired results. Effect of purity, surface conditions, preferred directions of growth, temperature gradient in melt, and other factors are considered.

G. M. Sinclair, USA

2796. Ramberg, W., and Miller, J. A., Twisted square plate method and other methods for determining the shear stress-strain relation of flat sheet, *J. Res. nat. Bur. Standards*, 50, 2, 111-124, Feb. 1953.

A method is presented for determining the stress-strain relation in shear for isotropic flat sheet. The method requires measurements of deflection or of extreme fiber-bending strain in the center portion of a twisted square plate. The octahedral stress derived from the stress-strain curve in shear for a twisted plate of aluminum alloy agreed within about 5% with the octahedral stress derived from tensile and compressive tests of the same material. The difference was much greater for specimens of mild steel with a definite yield point. Unfortunately, the method is difficult to perform on thin sheet because of the small size of the specimen. An examination of alternate methods indicates particular promise for the use of an annulus of constant thickness. From authors' summary by R. E. Heninger, USA

2797. Seemann, H. J., and Bentz, W., Ultrasonic techniques in testing of cylinders and steel bottles (in German), *Arch. Eisenhüttenw.*, 24, 1/2, 47-52, Jan./Feb. 1953.

Ultrasonic pulses (1 or 3 mc/sec) are not only used for detecting defects in chilled roll iron, but also allow distinguishing fine-grained structures. Bombs for highly compressed gases are tested for flaws with a combined reflection-transmission

method; the horizontal or vertical bomb is totally or partially filled with water or carbon tetrachloride.

O. Ruediger, Germany

Mechanical Properties of Specific Materials

(See also Revs. 2726, 2773, 2781, 2788, 2790)

2798. Craemer, H., The interplay of concrete of different ages in structural members, *Struct. Engr.*, 31, 1, 24-28, Jan. 1953.

When existing concrete is reinforced with a sheath of new concrete placed around it and bonded to it, the stress distribution under loads later applied is considerably influenced by shrinkage and creep. These develop at different rates in the old and new sections. The new concrete carries less than its share of added load. Under some conditions the older concrete can be shown to pick up added stress because of the addition of the new concrete. It is noted that ultimate strength is always increased by the added concrete.

The practice of temporarily omitting a small segment of a two-span continuous beam at a support with the idea of reducing dead-load negative moments is shown to have reduced effectiveness because of the subsequent shrinkage and creep of the concrete.

The mathematical relationships used were developed by the author in earlier papers (in German).

P. M. Ferguson, USA

2799. Rao, K. L., Extensibility and cracking in concrete, *Trans. Fourth Congr. Inter. Comm. Large Dams*, New Delhi, Jan. 1951, in 4 vols., vol. III, pp. 249-269. \$48 per set.

Paper deals with the experimental study of the "extensibility" of mortar and concrete, i.e., the unit tensional strain at which the concrete cracks. By using prismatic and cylindrical tension, bending and torsion specimens, reinforced with one or several steel rods, author finds that the extensibility is much greater in bending than in tension and that it increases much with an increase in the percentage of longitudinal reinforcement from 1.2 to 2.5%. It increases also with the cement content, and the bending variety with the sand content. Prestressing of reinforcement naturally improves it greatly.

The reviewer notices a number of weaknesses in the investigation. Thus, the strain at which cracking occurs is supposed to be partly due to plastic flow, yet no details of speed and time of testing are given. It is not stated whether or not the specimens were dried before testing, and no reference is made to the all-important phenomenon of shrinkage. The peculiarity of dependence of extensibility on percentage of reinforcement and on the state of stress, whether tension or bending, is not explained, and may very well have something to do with the crudeness of the visual crack detection on the whitewashed surface.

A. Hrennikoff, Canada

2800. Revesz, S., The cracking load of a composite prestensioned T beam, *Civ. Engrg., Lond.*, 47, 555, 733-735, Sept. 1952.

The varying moduli of elasticity of two concretes in prestressed composite sections and their effect on the cracking load were analyzed theoretically and investigated experimentally. Strain observations were made on a 14-ft composite T-beam with a pretensioned web.

Strain observations were made in the center of the beam with an 8-in. mechanical extensometer (Metzger gage), from which strain diagrams were drawn and the position of the neutral axis determined. The ultimate strength of the concrete in the web was approximately 7800 psi, and that in the in-situ flange was approximately 2200 psi. From this investigation it was con-

cluded that the cracking load of the particular section investigated was not reduced by the existence of the varying moduli of elasticity of the two concretes nor the differential shrinkage between the in-situ and the precast concretes. It was suggested that the composite T-beams of the type tested may be designed in the conventional manner with no appreciable reduction of the cracking load because of the varying moduli of elasticity or because of the differential shrinkage. J. N. Thompson, USA

2801. Stanford, E. G., Study of recrystallization of aluminum, *Research, Lond.* 6, 1, 21-25, Jan. 1953.

Author describes how elastic vibrations have been used to determine the variation in the modulus of elasticity (dynamic) with temperature, and shows how the results obtained enable deductions to be made on the recrystallization characteristics of aluminum, namely, "(1) the temperatures of the onset and completion of recrystallization under given conditions of heating; (2) the interrelationship between time, temperature, and recrystallization; (3) whether or not the material is completely recrystallized after a given annealing treatment (if it is, then its modulus-temperature curve will exhibit no 'dip'); (4) the effect of cold work on the interrelationship between time, temperature, and recrystallization; (5) the effect of cold work on the texture of materials after recrystallization." Author feels that "this method of measurement provides an additional powerful tool for use in the field of physical metallurgy and that the results obtained can be applied directly to the study of recrystallization and, perhaps, precipitation-hardening processes." A. S. Andes, USA

2802. Jones, W. R. D., and Bartlett, W. L., The viscosity of aluminium and binary aluminium alloys, *J. Inst. Metals* 81, 145-152, 1952/1953.

Article gives method of evaluating the castability of metals by special viscometer. Authors give complete description of apparatus and method used, and claim accuracy ($\pm 0.5\%$) which is superior to that of former investigations. Authors' synopsis states: "An apparatus, based upon the outer-rotating-cylinder method, has been constructed to study the variation of viscosity with temperature for pure aluminum and its variation with temperature and composition for a number of binary aluminum alloys. The results show that aluminum and its alloys exhibit a characteristic change point on their viscosity-vs.-temperature curves at 760-770 C, which appears to be due to some change of state in the liquid. For the binary alloys, the viscosity varies in accordance with the equilibrium diagram, there being a maximum at the limit of solid solubility and a minimum at the eutectic. Modification increases the viscosity of the eutectic aluminum-silicon alloy by about 30%, and either eliminates or raises the temperature of the change point on the viscosity-vs.-temperature curve." A. S. Andes, USA

2803. Brenner, P., and Metcalfe, G. J., The effect of cold work on the microstructure and corrosion-resistance of aluminum-5% magnesium alloys containing 0.1% zinc, *J. Inst. Metals* 81, 261-268, 1952/1953.

The effects of cold work, of prolonged aging at 70 C, and of additions of up to 1% zinc on the microstructure, corrosion resistance, and stress-corrosion resistance of an aluminum alloy containing 5% magnesium have been investigated.

The presence of zinc has little effect on the mechanical properties, but the alloy containing 1% zinc shows pronounced precipitation after aging at 70 C, and corrosion attack is greater than in the case of the 0.05% zinc alloys, which show an incomplete grain-boundary network after aging.

Small amounts of cold work reduce the corrosion resistance of the alloys appreciably, but heavy cold work both of aged and unaged materials results in a corrosion resistance as high as that of unworked material. The maximum corrosion attack, at about 30% cold work, is associated with the presence of continuous grain-boundary films formed on aging. Comparable behavior has been found in stress-corrosion tests, the maximum susceptibility to stress corrosion occurring in material that has been given 30% cold work. Heavy cold work (50% reduction in thickness) produces material of high stress-corrosion resistance. Explanations of the corrosion behavior are given.

From authors' summary

2804. Metcalfe, G. J., Atmospheric corrosion and stress-corrosion of aluminium-copper-magnesium and aluminium-magnesium-silicon alloys in the fully heat-treated condition, *J. Inst. Metals* 81, 269-278, 1952/1953.

The corrosion behavior of the aluminum alloys H10-WP and H15-WP in the extruded form has been determined in the stressed and unstressed condition by exposure to sea water, river water, and various natural atmospheres. The corrosion attack was assessed by visual and microscopical examination and by tensile tests on the corroded material. The most severe attack of both alloys resulted from exposure to the industrial atmosphere of Sheffield, where the average loss of strength after two years' exposure was approximately 11%, which is equivalent to a loss of thickness of 0.012 in. There was no indication of stress-corrosion failure of either of the alloys at any of the exposure sites. The high stress-corrosion resistance of the H15-WP alloy is somewhat surprising, since in sheet form the alloy is known to be very susceptible to stress-corrosion failure. The absence of such failure is attributed to preferential attack ("foliation") occurring along grain boundaries and bands parallel to the direction of extrusion which redistributes the concentration of stress at corrosion pits. The rate of loss of strength of both alloys exposed at Sheffield, and of H15-WP alloy exposed to a marine atmosphere, was found to decrease with time, apparently exponentially. Scatter of the results from the remaining sites was appreciable, but it was clear that, in general, there was a decrease in the rate of corrosion with time.

From author's summary

2805. Toft, Fl., Materials for refractories (in Danish), *Ingén Vidensk. Skr.* no. 1, 184 pp., 1952.

Refractory materials present one of the principal problems for the performance of chemical and metallurgical processes requiring high temperatures. Demands on thermal resistance and long life have steadily increased. Book deals specifically with refractory materials for gas works, but at the same time refers also to other similar industries.

First chapter gives a detailed survey on the properties of refractory materials generally used. Attention is concentrated on silica and fire-clay materials. Also discussed are insulating materials, common bricks, concrete, and steel.

Following chapter deals with the design of the oven and the right choice of refractory materials. Data on heat transfer and thermal conductivity of various materials are plotted in charts. Special attention is given to the design of walls and arches.

Other chapters deal with testing specifications and temperature measurements.

From author's English summary

2806. Dannöhl, W., Development and status of permanent magnetic materials (in German), *Stahl u. Eisen* 73, 2, 65-81, Jan. 1953.

Author presents extensive review and evaluation of permanent magnetic materials for the past fifty years.

F. J. Mehringer, USA

2807. Monzée, G., Glass as a structural material (in French), *Rev. Univ. Min.* (9) 9, 1, 3-14, Jan. 1953.

Much of the article is devoted to general discussion of structure and properties of glass which have only indirect bearing on the subject. Theoretical discussion of mechanical strength is good; but the effect of the unusual strength characteristics of glass on engineering application is not developed. Examples of use of glass are cited without consideration of structural principles and problems involved.

E. B. Shand, USA

2808. Peukert, H., Tensile strength and fracture resistance of molded Plexiglas M 33 (in German), *ZVDI* 95, 5, 119-122, Feb. 1953.

Article deals with the tensile and impact strength of thermoplastic material Plexiglas as depending on the deformation rate in molding process at elevated temperature. The tests were carried out at room temperature. Author shows that a true and distinct transition range of the deformation rates causing increased strength values does exist. The explanation of this phenomenon is based upon the assumption that in unmolded or very little molded material the molecular chain structure is disorderly distributed, and thus the material can give only a limited resistance against stress. When highly molded material is subjected to stress in the molding direction, the strength is expected to be far greater, as in this case the molecular forces must be considered to contribute to the total strength in quite another way, especially if the molecular chains are parallel to the direction of principal stress. Studies of rupture surfaces and distributions of permanent strain in broken test pieces seem to confirm this theory.

R. Nilson, Sweden

Mechanics of Forming and Cutting

2809. Chao, B. T., and Trigger, K. J., The significance of the thermal number in metal machining, *Trans. ASME* 75, 1, 109-115, Jan. 1953.

By a theoretical investigation, the temperature distribution in a workpiece during orthogonal cutting operations has been studied. The necessary mathematical simplifications are appropriate to the problem and the results are fit for practical purposes. Most interesting is the introduction of a special parameter, the "thermal number," which allows many technically important features to be described by this parameter only; e.g., effect of cutting speed, specific energy of metal removal, etc. Paper gives reference terms for economy in metal-cutting processes.

As is pointed out in R. S. Hahn's discussion, the single parameter statement requires further investigation with respect to the dependence of the thermal number on the thermal conductivity.

F. Wever, Germany

2810. Thompson, F. C., Carroll, J. B., and Bevitt, E., The drawing of steel wire at elevated and sub-normal temperature, *J. Iron Steel Inst.* 173, part 1, 36-51, Jan. 1953.

While hot-drawing is a standard process for tungsten, molybdenum, and stainless-steel wires, tests carried out for drawing at subnormal temperatures have not been reported hitherto. For the hot-drawing tests, three types of machines were used, one a dead-load, another a live-load machine. The third one was a high-speed machine with speeds up to 34 fpm. The heating arrangement and the dynamometer are described in detail. The first requirement was to find a suitable lubricant. A dispersion of semi-colloidal graphite in white spirit proved excellent up to

625 C. Even higher temperatures may be attained. A lime coat proved useful. The plots of drawing load vs. drawing temperature do not follow a simple course, as three components enter; the frictional component increases hyperbolically with the temperature. In the same way, the hardness of the metal decreases. However, the aging effect has a maximum at a certain temperature, resulting in a wave-shaped characteristic. This means that relatively low drawing loads are encountered at low as well as at high temperatures and an increase of drawing load at a medium temperature.

The mechanical properties of hot-drawn mild steel wire changed with the aging. While drawn at low temperatures the maximum stress increases; it is reduced when drawn at high temperatures. Similar observations were made with the elongation. With regard to the test at low temperature, little difficulty was found in drawing mild and carbon steel wires at temperatures down to -40 C, a suitable lubricant assumed. The lubricant was applied to the wire at normal temperature and then wire and lubricant were cooled together. The lowering of temperature caused an increased drawing load irrespective of the lubricant, due probably to the stiffening of the wire. The increase in drawing load for a given temperature drop was of the same order with various lubricants, and the most efficient lubricants at room temperature (soaps) remained so at lower temperature. No heavy reductions as at normal temperatures may be performed at low temperatures, as the drawn wire reverting to normal temperature is not capable of supporting the increased load. However, perfectly good wire can be produced at temperatures down to at least -40 C.

P. Grodzinski, England

2811. Geleji, A., Graphical determination of drawing schedules for tubes (in German), *Acta Techn. Hung.*, Budapest 4, 1-4, 347-363, 1952.

Article deals with the determination of drawing schedules for the cold drawing of tubes based on mathematical data developed by the author, using also the work of G. Sachs, F. Koerber, and A. Eichinger. A complex formula has been developed for the permissible reduction dependent on the degree of deformation reached for electrolytic copper, brass, and aluminum, in conformity with the reduction of the cross-sectional area of the pipe and assuming three values of wall friction, i.e., $f = 0.05, 0.1$, and 0.15 . A nomogram for the determination of the drawing forces has been developed. A practical example is given for the drawing of a brass tube (Ms 63) in the annealed condition.

P. Grodzinski, England

2812. Hill, R., A note on the back-pull factor in strip-drawing, *J. Mech. Phys. Solids* 1, 2, 142-145, Jan. 1953.

An expression is derived for the back-pull factor in strip-drawing through a rough wedge-shaped die, based on the slip-line field theory of Hill and Tupper. The range of reduction considered is that corresponding to uniform pressure on the die face. The drawing load for a given coefficient of friction and back pull may be computed from the back-pull factor in conjunction with the drawing load for a rough die without back pull [Green and Hill, title source, 1, 31, 1952]. It is shown that the currently accepted formula of MacLellan, based on the approximate theory of Sachs, is appreciably in error when the reduction or the coefficient of friction is large.

J. F. W. Bishop, Scotland

2813. Feldman, H. D., Investigations on the force and energy requirements in the cold-flow pressing of different types of steel (in German), *Stahl u. Eisen* 73, 3, 165-174, Jan. 1953.

Author points out that the application of cold-flow pressing to steel, a rather recent art, is limited by the high force require-

ments. His investigation therefore attempts to determine the force requirements, through experiment and calculation, for the principal types of flow-press application. The three types are categorized as follows: (1) Blank is a cylindrical cup, material flows in direction opposed to ram motion; (2) blank is cylindrical cup, material flows in direction of ram motion; and (3) blank is piece of rod, material flows in direction of ram motion. Experimental force and energy data for a particular process and steel were obtained with several tests varying in terminal reduction of area. The force was measured with a special device and input-energy diagrams were produced. A grid was applied to the plane faces of specimens split through their longitudinal axis, which, along with Vickers hardness readings, facilitated deformation studies.

This experimental work, plus the use of well-known plasticity relations for calculating force and energy requirements, led to some interesting results. Among the more important were: (1) The average efficiencies, based on the ratio of calculated flow energy to measured input energy, were found to lie between 35 and 85% for the three processes investigated; (2) the average flow-press velocity was found to influence the maximum flow-press force in a manner dependent on the magnitude of the reduction of area; and (3) an approximate relation between maximum force and volume of displaced material, parabolic in nature, was shown to be common to all three processes for each of several particular steels. The last figure in the paper presents a nomogram method for calculating the maximum flow-press stress and total input energy for a particular total value of the principal deformation.

J. Miklowitz, USA

2814. Collaud, A., Investigation on tool steels of great cutting capacity (in French), *von Roll Mitteilungen* 11, 3/4, 73-91, July-Dec. 1952.

The application of heat treatment together with a skillful selection of alloying elements such as tungsten, molybdenum, and vanadium produces steels for very high cutting speeds. These steels are superior in many respects to quality 18/4/1, owing to their considerably lower tungsten contents. When they are properly composed and heat treated, the objections made against them with regard to the retention of hardness of the martensite and to the hot hardness appear to be unfounded. It must be noted that, whatever the addition to tungsten may be, steels of high chromium content retain the typical character of chromium steels as far as their toughness and the stability of the austenite and the martensite are concerned.

Steels of high chromium content can be forged perfectly, rolled and cast, thus increasing considerably the possibility of their use for tools of a high cutting capacity.

From author's summary by A. O. Schmidt, USA

2815. Boston, O. W., A standard of procedure for evaluating the tool life of single-point sintered carbide tools, *Ann. Meet. ASME*, New York, Dec. 1952. Paper 52-A-39, 10 pp.

Paper presents a proposed American standard. Method consists of measuring width of wear land on tool flank using a microscope. End point for stating tool life is normally taken as 0.030-in. width of wear land. Flank wear is found to be a much more readily measurable indication of tool life than face wear. Method may be used to evaluate relative performance of different work materials, cutting tools, cutting fluids, or machining conditions. In so doing, tests are usually run at several different cutting speeds to obtain the corresponding values of tool life based on 0.030-in. flank wear. The results, plotted on log-log coordinates, normally follow the classical Taylor exponential law.

These serve to evaluate the parameters in that relationship for each condition tested, thus evaluating relative performance.

M. E. Merchant, USA

2816. Cook, M., and Richards, T. Ll., *Fundamental aspects of the cold working of metals*, London, Inst. Metals, "The cold working of non-ferrous metals and alloys," 7-26, 1952. \$2.50.

Reference is made to the nature of the metallic state and metallic cohesion in terms of the electron theory of metals, and a brief outline is given of the several mechanisms involved in plastic deformation. These include crystallographic slip, twinning, and kinking, and a shear mechanism to which particular attention is drawn because of its importance in many metal-fabrication processes. The influence of plastic deformation on structure is described with special reference to the development of preferred orientation in view of its technological significance, while the effect of deformation on the fine structure, as revealed primarily by x-ray diffraction studies, is considered in terms of the dislocation theory. The relation of work-hardening and plasticity to the structural changes brought about by cold-working is also discussed.

From authors' summary by L. V. Colwell, USA

2817. Davies, C. E., *The cold rolling of non-ferrous metals in sheet and strip form*, London, Inst. Metals, "The cold working of non-ferrous metals and alloys," 45-80, 1952. \$2.50.

After reviewing recent progress in rolling practice, the author describes modern technique in rolling (a) copper and copper alloys, and (b) aluminum and its light alloys. The relative merits of two-high and four-high mills for various purposes are described, and also those of reversing, nonreversing, and tandem units. Reference is made to the latest designs for mills with minimum work-roll diameters, such as the Sendzimir mill. Considerable attention is given to the auxiliary equipment required to handle sheet and strip during rolling.

From author's summary by L. V. Colwell, USA

2818. Cleaver, F. T., and Miller, H. J., *Wire-drawing technique and equipment*, London, Inst. Metals, "The cold working of non-ferrous metals and alloys," 81-106, 1952. \$2.50.

The principal landmarks in the development of the wire-drawing industry in this country from the earliest beginnings until the present time are reviewed. A description is then given of present-day machines, comprising tandem- and cone-type machines, in which slipping of the wire occurs in the course of drawing, and also the nonslip variety. Die design, die materials, lubricants, speeds of drawing, reductions, and other aspects of wire drawing are dealt with, and a detailed account is given of current practice in the production of copper, brass, bronze, and other copper-alloy wires, and also of aluminum and alloy wires. Finally, various types of defects which are encountered are considered.

From authors' summary by L. V. Colwell, USA

2819. Jevons, J. D., *The deep drawing and pressing of non-ferrous metals and alloys*, London, Inst. Metals, "The cold working of non-ferrous metals and alloys," 107-163, 1952. \$2.50.

The terms "sheet and strip" and "deep drawing and pressing" are commented on, and various methods of deep drawing and pressing are described. Crank and hydraulic actuation of presses are compared, and attention is drawn to certain merits of multipunch presses.

Tool materials and drawing lubricants are classified, and the usefulness and limitations of each are explained. Interstage annealing is described with particular regard to the faults com-

monly experienced under industrial conditions and to the incidence of critical-strain crystal growth.

The properties of sheet which determine its behavior under the press are discussed, and a nice balance between "tenacity" and "ductility" is suggested as being of primary importance in most instances. A number of ordinary and special tests applicable to sheet are described, and their limited usefulness in predicting behavior under the press is explained. A method for the routine acceptance testing of sheet under industrial conditions is suggested. The phenomena of stretcher-strain markings and of season cracking are reviewed and discussed.

From author's summary by L. V. Colwell, USA

Hydraulics; Cavitation; Transport

(See also Rev. 2763)

2820. Blackburn, J. F., Contributions to hydraulic control. 3—Pressure-flow relationships for a 4-way valve, Ann. Meet. ASME, New York, Dec. 1952. Paper 52—A-42, 15 pp., 7 figs.

Paper covers the static characteristics of spool and flapper-type 4-way hydraulic valves. Five cases are considered: Ideal (zero lapped) spool valve operating from constant pressure source, underlapped spool valve operating from constant pressure or constant flow source, and flapper valve operating from constant pressure or constant flow source. Operation from constant flow source is shown to be impractical from the standpoint of leakage flow and linearity. Although the ideal valve cannot be realized physically, due to radial clearance and imperfect spool edges, either an underlapped or overlapped spool valve operating well outside the region of lap may be considered ideal. A close approximation of the ideal valve operating from a constant pressure source is required for high power applications. Both underlapped spool valves and flapper valves operating from a constant pressure source find their greatest use in pilot-valve service.

Nondimensionalized load-pressure load-flow curves for various valve displacements are presented for each of the five cases without any linearizing assumptions being made. A table of valve characteristics is presented, the two most important characteristics being pressure sensitivity and flow sensitivity. These are defined as the output pressure-differential valve-displacement gradient with the valve centered and infinite load (motor blocked), and the output-flow valve-displacement gradient with the valve centered and zero load (output bypassed), respectively.

Much useful information of a practical nature is included in paper. Paper is recommended to anyone interested in hydraulic controls, although much of the material in the paper should already be familiar to hydraulic servo designers. For a treatment of the dynamics of 4-way valve servos, the reader is referred to AMR 6, Rev. 731.

S. Z. Dushkes, USA

2821. Raynaud, J.-P., Study of currents of muddy water through reservoirs (in French), *Trans. Fourth Congr. inter. Comm. Large Dams*, New Delhi, Jan. 1951, in 4 vols., vol. IV, 139-161. \$48 per set.

Report gives account of a first group of experiments undertaken at the Laboratoire Dauphine d'Hydraulique on the flow of currents of muddy water near the bottom of a reservoir containing still clear water. Experiments were made with currents of relatively low density (about 1.030) and for small Reynolds numbers (laminar and transition flows; for laminar flows a law was found of the usual type between λ (coefficient of loss of head) and Re , the constant depending perhaps on the particular muddy water employed. Author calculates tangential stresses for the areas of contact with the bottom and between muddy

and clear water, the velocity at this surface of separation, and the velocity distribution.

D. Citrini, Italy

2822. Szigyártó, Z., Dimensioning of storm water overflow outlets for city sewers (in Hungarian, with Russian and English summaries), *Vízügyi Közlemények* no. 2, 248-281, (46) (47), 1952.

Author discusses the problems in determining the size of storm water overflows for combined sanitary sewer and storm sewer systems. The biochemical aspects of the problem are treated in great detail, and a design procedure for such systems is given from the hydraulic point of view. Design details of components are also presented.

A. Fejer, USA

2823. Ihrig, D., Methodical marking of navigation channels on rivers, with specific reference to the Danube (in Hungarian, with Russian, German, and French summaries), *Vízügyi Közlemények* no. 2, 179-213, (36) (44), 1952.

For the navigation commission of the eight states bordering the Danube through its 1600-mile navigable course, author develops principles of proper marking of different river sections of slow, medium, and high velocity flow, deep and shallow, wide and narrow, straight and tortuous channels; also number and arrangement of towed barges, and type, size, shape, distance, and maintenance of markings and lights—all to speed up and increase freightage.

A. Hollander, USA

2824. Binnie, A. M., The stability of the surface of a cavitation bubble, *Proc. Camb. phil. Soc.* 49, part 1, 151-155, Jan. 1953.

Brief theoretical study is made of stability of surface of a cavitation bubble. Surface tension is both neglected and considered, and is found to have a marked stabilizing effect. Attempt is made to verify, by this theory, the experimental work of others.

E. F. Macks, USA

Incompressible Flow: Laminar; Viscous

(See also Revs. 2702, 2873, 2885, 2964)

2825. Dommasch, D. O., Principles of aerodynamics, New York, Toronto, London; Pitman Publ. Corp., 1953, xvii + 386 pp. \$7.50.

This book has been written primarily to present the basic physical laws and methods used to apply these laws to specific problems. The author makes no claim to mathematical rigor, and thus the presentation of the basic laws is made in a simple direct manner suitable for use in an elementary course in aerodynamics. The methods used to apply these basic laws to specific problems or classes of problems are discussed very thoroughly, with few or no steps omitted for the reader's exercise.

In the first chapter, the general concepts of continuity of mass and momentum (Euler equations) are derived from very elementary considerations for frictionless fluids. The basic considerations are concluded with some classical explanations of energy and quantities such as entropy, enthalpy, and specific heat for perfect gases.

The remainder of the book is devoted to an introduction of mathematical techniques for handling two- and three-dimensional, compressible and incompressible-flow problems.

One chapter is devoted to complex variables and vectors for the aid of those who may be unfamiliar with these subjects. This chapter presents only the most elementary concepts of the subjects discussed.

The final chapter is devoted to "Some effects of viscosity." Once again only the classical treatment is presented, and far too little is said regarding the limitations of the resulting equations.

The book is well written and the publishers have done an excellent job. Its use, however, must be limited to very elementary courses in aerodynamics, due to its lack of rigor. The greatest contribution it can claim is that it has presented the basic facts regarding classical, compressible and incompressible fluid flow in two and three dimensions in one volume, with consistent notation and complete mathematical derivations of the methods used for solution of problems in each type of flow.

M. Alperin, USA

2826. Michael, W. H., Jr., Flow studies in the vicinity of a modified flat-plate rectangular wing of aspect ratio 0.25, *NACA TN 2790*, 33 pp., Sept. 1952.

An investigation was made in order to study the characteristics of the flow in the vicinity of a rectangular wing of aspect ratio 0.25 with a modified flat-plate airfoil section. The investigation was conducted by means of photographs of a tuft grid located at a number of chordwise positions along the airfoil and behind the trailing edge of the airfoil. Supplementary measurements of the vorticity distribution in the wake were made with a yaw-head Pitot-tube installation.

The results indicate that there is a rapid rolling-up of the trailing vorticity along the chord of the wing and that, at the higher angles of attack, there is a distinct vortex visible at 12.5% of the chord. The slopes of the vertical locations of the vortex cores are predicted very well by the theory of Bollay.

The chordwise growth of lift calculated from the trailing-vortex strengths shows that the lift increases fairly rapidly across the forward part of the chord, with approximately 70 to 85% of the total wing lift present forward of the midchord position. The total lift calculated from the tuft-grid photographs showed good agreement with the lift measured on the balance system.

C. F. Bonilla, USA

2827. von Mises, R., On some topics in the fundamentals of fluid flow theory, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 667-671, 1952.

Paper is concerned only with mathematical questions. A study is made of the consequences which can be derived from a given set of differential equations without adducing any knowledge drawn from observation, etc. Paper considers five unknowns, the three components of the velocity vector q , the pressure p , and the density ρ . These are considered as functions of the four independent variables x, y, z, t . The first three differential equations are written as one vector equation (called Newton equation). The continuity equation is then written, and also an additional fifth equation which specifies the particular type of flow to be studied. A study is then made of discontinuous solutions, ideal fluid flow, viscosity and heat conduction, and shock phenomena.

C. B. Matthews, USA

2828. Koning, C., Some interference problems, *Anniv. Vol. appl. Mech.*, dedicated to C. B. Biezeno; Haarlem, Antwerpen, Djakarta, N.V. De Technische Uitgeverij H. Stam, 1953, 82-98, Hfl. 20.

The type of interference problems referred to in this paper deals with a body of arbitrary shape which is placed in a stream of fluid. When the body is of a complicated shape, it is customary to consider the body as being split up into several elementary shapes (i.e., wing, fuselage, propeller), and then to determine the properties of each elementary shape when placed in a uniform stream of fluid. An interference problem occurs

when at least two such elementary bodies are both situated in the same stream of fluid such that the flow field of each body is disturbed, due to the presence of the other bodies. Very often interference problems are so complicated that exact theoretical solutions are not readily obtainable.

In this paper a special group of interference problems is considered, which can be idealized as an "actuator disk" in the vicinity of a streamline body. Both the streamline body and actuator disk are situated in a uniform parallel flow of an incompressible and frictionless fluid. The actuator disk is a rough approximation of a propeller and is defined as a plane disk normal to the undisturbed flow where the fluid passing through it gets a pressure drop or rise which is uniformly distributed throughout the disk. Momentum and energy equations are used to characterize the relationship between the forces on the actuator disk and the streamline body.

J. Aronofsky, USA

2829. Slezkin, N. A., Plane flow of an ideal fluid about a gas-filled shell (in Russian), *Uchen. Zap. mosk. Univ.* 152, *Mekhanika* 3, 61-75, 1951.

The nonrigid gas-filled shell is assumed to be convex and to be under constant tension. External forces are not treated. The boundary condition on the surface of the body becomes $a + bV^2 = r_1^{-1} + r_2^{-1}$, where r_1 and r_2 are the principal radii of curvature, a and b are constants, and V is the velocity. The two-dimensional case is treated in detail. In this case, the problem may be reduced to the solution of the following integral equation

$$T(\beta) = -\lambda \int_0^\pi [e^{T(\alpha)} \sin^2 \alpha + ke^{-T(\alpha)}] \ln [4 \sin^2 (\alpha - \beta)] d\alpha$$

A process of successive approximations is introduced and proved convergent to a solution if the nondimensional parameter λ is below a given bound (giving a relation between the arc length, the velocity at infinity, the tension, and the density). An approximate computation is carried out, giving an elliptical cross section with the long axis perpendicular to the flow.

J. V. Wehausen, USA

2830. Ehrich, F. F., Penetration and deflection of jets oblique to a general stream, *J. aero. Sci.* 20, 2, 99-104, Feb. 1953.

Paper treats problem of smooth entry of two-dimensional jet into passing stream as first step in penetration and mixing studies of complex systems. Solutions are obtained, for systems with straight-sided jet slots inclined at 90° and 180° to main stream, by standard free-streamline potential-flow methods. Author concludes that only one jet velocity corresponds to each main flow velocity for a particular geometry. Velocity of smooth entry should be avoided if mixing is desired. Reviewer believes author's comparison with circular orifice penetration data shows derivation in correct direction. Paper is of interest in boundary-film insulation and mixing problems.

A. D. K. Laird, USA

2831. Szebehely, V. G., Generalization of the dimensionless frequency parameter in unsteady flows, *David W. Taylor Mod. Basin Rep.* 833, 26 pp., Nov. 1952.

The complex and relatively unknown field of time-dependent hydrodynamic phenomena is approached from a general point of view. Only a few special flows are discussed, emphasizing the diversity of unsteady-flow problems. Since time effect occurs only in the acceleration term of the momentum equation, an analysis of the two types of acceleration is presented in detail. A dimensionless ratio of the local and convective accelerations (generalized Strouhal number) is introduced. It is shown that

with the magnitude of this ratio the unsteadiness of the flow can be described and characterized. A short discussion of accelerationless flows (zero acceleration at any time at any point) is included.

From author's summary by H. N. Abramson, USA

2832. Reichardt, H., and Tollmien, W., The distribution of the flow volume in a given branching system (in German), *Mitt. Max-Planck-Inst. Strömungsforschung* no. 7, 48 pp., 1952.

Publication deals with fluid flow in branched systems as may be found in manifold applications. The problems of influx and efflux are discussed for a single manifold having numerous branches, and for a pair of manifolds connected by branching ducts. The latter problem is considered for unflow and for reversed flow.

An attempt is made to correlate special experimental results with conclusions drawn from the Bernoulli equation. Data are plotted in dimensionless form for three cases, namely, $f/b = 0.5$, $f/b = 1.0$, and $f/b = 2.0$. In this ratio, f is the total width of the branches while b is the width of the manifolds. Rectangular cross sections are used. It is concluded that, for efflux from a single manifold, Bernoulli's equation is valid. For influx into a single manifold, an approximation is suggested which incorporates the effect of losses. For a double manifold system, it is shown that the conditions in the collector are the governing criteria of the flow.

Publication should be of value in applications of manifolds or other types of branched flow.

A. B. Cambel, USA

Compressible Flow, Gas Dynamics

(See also Revs. 2825, 2827, 2874, 2918)

2833. Wu, T. Y., and Cole, J. D., Anemometry of a heated flat plate, *Heat Transf. Fluid Mech. Inst.*, Stanford Univ. Press, 139-158, 1952. \$4.

Some results of linearized theory of compressible viscous and heat-conducting gas, and application of theory to the problem of heat loss from two-dimensional heated plate are presented. Linearized system of equations facilitates over-all picture which is usually qualitatively correct. Dependence on Mach, Reynolds, and Nusselt numbers can easily be studied. Errors of definite type can be estimated and quantitative information obtained.

Problem of heated flat plate was suggested by use of hot-wire anemometers at high subsonic and supersonic speeds. Reynolds number of wire based on diameter is about 20. This is not low enough for Stokes' flow-type solution nor so high that some corrections to boundary-layer theory might not be important. Corrections are estimated using linearized theory.

Fundamental solution of linearized equations shows the flow field composed of a diffusive part, the wake and a dynamic part, which behaves like a nonviscous outer flow for $M < 1$. For $M > 1$, dynamic part can still be thought of as nonviscous flow past a fluid source, and flow field consists of certain Dirac delta functions on nonviscous Mach lines.

Application of theory to problem of heated plate shows two types of behavior, depending on Reynolds number. For Re large, first-order term of Nusselt number is proportional to $Re^{1/2}$, but is independent of Mach number. Second-order term is independent of Re but shows explicit dependence on M . Results are qualitatively in agreement with experiments on hot wires in that, if wire is heated sufficiently, changes in M have small effect on heat transfer. Quantitatively, when $M = 0$, result of presented theory is about 30% higher than result of Fage's experiment, while Fage's and Tsien's theoretical results are 18% and

12% too low, respectively. For Re small, linearized theory shows that Nu depends on Re only, and Mach number effect drops out.

From authors' summary by J. Jerie, Czechoslovakia

2834. Ertel, H., A theorem on asynchronic-periodic vortex motions of compressible fluids (in German), *Miscellanea Academica Berolinensia*, I, 62-68, Akademie-Verlag, Berlin, 1950.

The author considers steady barotropic motions of an ideal fluid subject to conservative extraneous force, under the special assumption that all streamlines are closed curves. The period of rotation is then $\tau = \tau(a, b, c)$, where a, b, c are the material coordinates. Let $H(a, b, c)$ be the usual Bernoullian expression, and put $L = H + v^2$. Put $W = \oint L dt$, where the integration is carried out for a fixed particle. By integrating the material equations of motion over one period, the author derives the theorem $\tau = dS/dH$, where $S = W + H\tau$. This is the analog of a proposition in the Hamilton-Jacobi theory. The author shows that S is in fact the circulation around the closed streamline, and thus, since this streamline must lie upon a surface $H = \text{const}$, the theorem can be interpreted in kinematical terms.

C. Truesdell, USA

2835. Flanagan, L. E., Jr., Comments on "Two-dimensional airfoils at moderate hypersonic velocities," *J. aero. Sci.* 20, 2, 150-151, Feb. 1953.

In this Readers' Forum article of the *IAS Journal*, author has compared equations derived by Dorrance with expressions developed by Miles and Busemann for power series for pressure coefficients of expansive and nonisentropic compressive-flow conditions at hypersonic speeds (defined as the range where $(M^2 - 1)^{1/2}/M \cong 1$). He proceeds to point out that these expressions, while derived by different methods, are identical up through and including the third-order terms, and that values beyond this are unimportant, inasmuch as the series converges rapidly. This agreement is important and this note is considered a useful comment on this subject.

E. A. Bonney, USA

2836. Moore, F. K., Unsteady oblique interaction of a shock wave with a plane disturbance, *NACA TN 2879*, 66 pp., Jan. 1953.

Consider a flow in which a plane normal shock is propagating into a medium at rest. Author investigates small perturbations of this flow due to various plane disturbances with arbitrary profile. Among the cases analyzed are the flows due to plane sound waves impinging upon the shock from various directions, and that due to the shock overtaking a stationary plane vorticity wave of constant pressure. The resultant interaction in all cases except where the shock overtakes either the sound or vorticity wave from certain directions may be expressed in terms of plane vorticity and sound waves; in the exceptional cases, the plane sound wave is replaced by an attenuating isentropic pressure wave. Computations for the case of the interaction of the shock with the sound waves are carried out for Mach numbers 1, 1.5, and ∞ for various incidences of the sound wave.

H. Yoshihara, USA

2837. Truesdell, C., On curved shocks in steady plane flow of an ideal fluid, *J. aero. Sci.* 19, 12, 826-828, Dec. 1952.

For an inviscid fluid obeying an arbitrary equation of state, the jump in the velocity gradient across a curved shock in steady plane flow is calculated. The results are expressed in terms of dimensionless variables. For the case of uniform oncoming

flow, it is shown that the vorticity generated by the shock is a function of curvature, strength, and shock angle only, being independent of the form of the equation of state.

From author's summary by L. M. Milne-Thomson, England

2838. Bull, G. V., Fowell, L. R., and Henshaw, D. H., *The interaction of two similarly facing shock waves*, *Univ. Toronto Inst. Aerophys. Rep.* no. 25, 27 pp., 6 figs., Jan. 1953.

Computational results are given for various strengths of the interacting shocks and for several values of the ratio of specific heats.

H. Yoshihara, USA

2839. Krzywoblocki, M. Z. E., *Bergman's linear integral operator method in the theory of compressible fluid flow*, *Öst. Ing.-Arch.* 6, 330-360, 1952.

In the first part of his work on Bergman's linear integral operator method, author discusses subsonic flow. After deriving Chaplign's equation, author thoroughly discusses the fundamentals of Bergman's method, the duality between the flows of an incompressible and a compressible fluid, as well as the problem of the convergence of the series expansion. A brief representation of the simplified pressure-density relation according to von Mises' proposition closes this part. From author's summary

2840. Truitt, R. W., *A method for location of detached shock waves ahead of plane bodies*, *J. aero. Sci.* 20, 1, 61-62, Jan. 1953.

Author takes vigorous exception to prediction of Moeckel [AMR 3, Rev. 2406] that distance of detached shock wave ahead of sonic point of a wedge divided by thickness of wedge at sonic point is independent of wedge angle. To prove his point he develops expression showing that distance of detached shock wave from leading edge of wedge, divided by length of wedge from leading edge to sonic point, depends explicitly on wedge angle. This in no way contradicts Moeckel's result. In fact, both authors make the same basic assumptions and arrive at the same results, as can be demonstrated easily. Reviewer believes author was confused as to contents of Moeckel's paper.

R. E. Duff, USA

2841. Gould, D. G., *The head-on collision of two shock waves and a shock and rarefaction wave in one-dimensional flow*, *Univ. Toronto, Inst. Aerophys. Rep.* no. 17, 28 pp., 2 tables, 34 figs., May 1952.

In the first part, author discusses wave interactions mentioned in title, using Rankine-Hugoniot equations. Nature of waves after collision is determined with help of a graphical analysis in pressure-velocity plane, as outlined by Courant and Friedrichs ["Supersonic flow and shock waves," Interscience Publishers, 1948]. Algebraic equations are obtained for final shock strengths. Results are used, in combination with a method of Etkin [AMR 5, Rev. 1464], as basis of a finite difference method for calculating flow during collision of shock wave and rarefaction wave. Method is applicable also to other interactions.

Shock-tube experiments by author are described, and final shock strengths are compared with calculated values. In collisions of shocks with rarefaction waves of medium strength, good agreement was found up to shock pressure ratios of about 7. In collisions of two shock waves, initial pressure ratios were varied over range of about 31. Final pressure ratios agreed with predictions from Rankine-Hugoniot equations in first half of range. At higher shock strengths, agreement with calculated results was obtained when excitation of molecular vibrations was taken into account, using results of Bethe and Teller [Aberdeen Proving Grounds, BRL Rep. no. X-117].

A. Van Tuyl, USA

2842. Ribner, H. S., and Arnoff, E. L., *Interaction between a supersonic stream and a parallel subsonic stream bounded by fluid at rest*, *NACA TN* 2860, 45 pp., Dec. 1952.

In a laminar boundary layer subjected to an oblique incident shock wave a "dead-air" bubble appears. The boundary layer is simplified by a uniform subsonic stream bounded on one side by a uniform supersonic stream [AMR 3, Rev. 2711], on the other by the fluid at rest of the bubble. If the shock were incident directly on the dead-air region without the intermediary of the boundary layer, it would be reflected as an expansion wave, and the boundary streamline has a corner. The presence of the boundary layer sandwiched between the outer supersonic flow and the dead-air region is found, in the vicinity of the shock, to scarcely modify the shape of this corner. The displacement of each interface from its undisturbed position and the pressure distribution along the upper interface are calculated for different Mach numbers in the supersonic and in the subsonic stream, and for different thickness of subsonic region. The influence of the two Mach numbers can be expressed by one parameter only; that of the thickness is given by geometrical similarity.

A. Betz, Germany

2843. Heinrich, G., *Energy transport in moving fluids* (in German), *ZAMM* 32, 286-288, 1952.

For a viscous compressible fluid, the author calculates the rate of change of flow of mechanical energy through an arbitrary surface element. [Since only part of the energy change can be accounted for by surface integrals, any such result is arbitrary. More general analysis has been given by the reviewer, *Phys. Rev.* (2) 73, 513-515, 1948.]

C. Truesdell, USA

2844. Jenkins, R., and Aronofsky, J. S., *Unsteady radial flow of gas through porous media*, *Ann. Meet. ASME*, New York, Dec. 1952. Paper 52-A-26, 5 pp.

Authors integrate numerically the equation that governs the transient pressure distribution within a reservoir in radial direction. The results are presented in graphs.

Y. H. Kuo, USA

2845. Roberts, R. C., *Unsteady flow of a gas through a porous medium*, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 773-776, 1952.

The governing nonlinear differential equation is derived, assuming the flow obeys Darcy's law, the flow is isothermal, and the viscosity, permeability, and porosity are constant. A step-by-step process for obtaining an approximate solution, which can be made as accurate as desired, is presented. Explicit general solutions are given for the one-dimensional flow in a finite sand-filled pipe which is closed on one end and has either constant velocity or constant flux on the other end. These solutions are evaluated for particular initial and boundary conditions. The method is also applicable to radial flow.

W. J. Wells, USA

2846. Oppenheim, A. K., *Generalized theory of convective heat transfer in a free-molecule flow*, *J. aero. Sci.* 20, 1, 49-58, Jan. 1953.

An analysis of convective heat transfer in free-molecule flow is given. Results are presented for the case of the flat plate, the sphere, and the cylinder. While the basic analysis has been taken from the work of other investigators (referenced by the author), the values of the various functions are tabulated and the results of the analysis may be easily applied to bodies made up of simple shapes.

G. Goodwin, USA

2847. McFadden, J. A., Initial behavior of a spherical blast, *J. appl. Phys.* **23**, 1269-1275, 1952.

At time $t = 0$ a sphere of gas at uniformly high pressure is allowed to expand suddenly into a homogeneous atmosphere. The author evaluates the early stages of the motion before the head of the rarefaction wave has reduced its radius by a large fraction. He works from the zero-order solution given by the plane "shock tube" case, and calculates a first-order correction for the initial effects of sphericity. He gives also in principle the method for obtaining higher-order corrections. A worked example, where a diatomic gas expands into a diatomic atmosphere, indicates (as one would expect) that after a short time the rarefaction wave is stronger and the shock wave weaker than in the plane case, so that the pressure distribution is everywhere lower and both the shock and the tail of the rarefaction move more slowly away from the initial interface.

M. J. Lighthill, England

2848. Thom, A., and Klanfer, L., The method of influence factors in arithmetical solutions of certain field problems, *Aero. Res. Comm. Lond. Rep. Mem.* 2440, 30 pp., Aug. 1946, published 1953.

Paper gives an extension to the squares method of solving certain field problems. The idea of influence factors is used. This enables certain problems to be solved without squaring and reduces the actual arithmetical work in others to a fraction of that otherwise required. As an example, the effect of the channel walls is calculated for the flow of a compressible fluid past given profiles in channels of different widths. A comparison is also made of the resulting blockage factor with the values given by the mass flow method described in previous work.

From authors' summary by G. V. R. Rao, USA

2849. Koski, W. S., Lucy, F. A., Shreffler, R. G., and Willig, F. J., Fast jets from collapsing cylinders, *J. appl. Phys.* **23**, 12, 1300-1305, Dec. 1952.

Experiments with plane and toroidal detonation waves applied to cylindrical and conical metal liners, leading into a long evacuated tube, revealed the usual penetrating jet and also a faster jet. Relation of the latter to charge, cavity, and liner dimensions and liner material is demonstrated. Maximum velocity of fast jets is related to atomic weight. Highest velocity observed was 9 cm/microsec for beryllium. Spectrographic and other evidence are given for gaseous nature of at least part of the fast jet.

J. Corner, England

2850. Prince, D. C., Jr., The study of compressible flow through cascades using a linear pressure-volume relationship, *J. aero. Sci.* **19**, 11, 782-783, Nov. 1952.

Author briefly reports an application of the von Kármán-Tsien method to compressible flow through cascades. An example is shown for which the hodographs are circles whose diameter and orientation vary with Mach number. The resulting variations of cascade geometry and air angles with Mach number are discussed, and sufficient indication of the method is given to enable it to be used as a basis for comparing cascade performance at differing Mach numbers.

W. R. Hawthorne, England

2851. Roy, M., Fabri, J., Siestrunk, R., and Le Grivès, E., Net thrust and specific consumption of supersonic ramjets (in French), *ONERA NT* no. 8, 19 pp., 1952.

Performance is calculated as a function of combustion end temperature, nozzle throat area, and fuel/air ratio. A normal shock diffuser is considered, and linearized theory gives the wave drag,

while thermodynamic tables provide the combustion properties. No off-design-point calculations are made; this, together with the lack of consideration of better diffusers which have been realized in practice and the somewhat oversimplified combustion considerations (combustion efficiency is always 94%), result in less useful calculations than are readily available in the American literature. However, the calculation scheme exhibited by the authors could be readily extended to include more refined and realistic considerations.

B. W. Augenstein, USA

2852. Linning, D. L., The adiabatic flow of evaporating fluids in pipes of uniform bore, *Instn. mech. Engrs. Proc. (B)* **1B**, 2, 64-75, 1952.

Two-phase flow in pipes is important in a variety of engineering applications. This paper considers a liquid in equilibrium with its vapor so that there is no heat transfer between the phases. Author considers annular, separated, and frothing flow; the possibility of plug and bubbling flow is pointed out.

The theoretical treatment is one-dimensional, but the velocity of the vapor and liquid phase differ. Experimental data for annular flow are given, from which the difference is found. The theory then predicts critical outlet temperature and relative velocity factors which agree reasonably well with experiment.

An interesting discussion and author's closure follow the paper. Reviewer believes this paper to be a valuable contribution which suggests many topics for future work.

W. Squire, USA

2853. Callaghan, E. E., and Serafini, J. S., Analytical investigation of icing limit for diamond-shaped airfoil in transonic and supersonic flow, *NACA TN* 2861, 18 pp., Jan. 1953.

By considering the wing to behave as a wet-bulb thermometer constructed of zero-conductivity material, the heat- and mass-transfer equations for a wetted wedge airfoil are solved and reveal that at high Mach numbers, although the forward portion of the wing is heated above 32 F by compression and frictional effects, the afterbody may be below 32 F due to the expansion of the air and incomplete "recovery." Even at $M = 1.4$, icing may thus occur. This reviewer points out that icing encounters at this speed should be exceedingly rare, but the given limiting Mach number appears correct.

M. Tribus, USA

2854. Van Dyke, M. D., A study of second order supersonic flow theory, *NACA Rep.* 1081, 23 pp., 1952.

Supersedes article reviewed in AMR 4, Rev. 1653.

Turbulence, Boundary Layer, etc.

(See also Revs. 2524, 2842, 2939)

2855. Batchelor, G. K., The theory of homogeneous turbulence, New York, London: Cambridge Univ. Press, 1953, xi + 197 pp. \$5.

This is the first of the Cambridge monographs in mechanics and applied mathematics. The subject is homogeneous turbulence, defined as turbulence in which the properties do not vary with position in the field, although variation with coordinate directions is possible. Isotropic turbulence, in which properties are independent of rotation of the coordinate axes, is a particular case discussed at length in the book. The book is not written from the historical viewpoint nor from the aspect of experimental procedure, but is a logical mathematical development starting from the equations of motion. Hypotheses and empirical relationships are not introduced or discussed until the rigorous developments have been presented. The book contains very little

new material but is an exhaustive compilation of recent work in the field and covers all material presented up to mid-1952.

In first few chapters, the mathematical description of the turbulent velocity field is presented. Velocity correlation and spectrum tensors are described and their properties discussed. Symmetry conditions and equation of continuity are applied. Also included are the simplifications introduced by isotropic turbulence. There follows a chapter describing the solution of some linear problems in which decay of the turbulence is not important. Consequently, these problems—the effect of the screen on isotropic turbulence and the effects of a sudden contraction in the stream—can be rigorously solved.

In chapter on dynamics of decay, Navier-Stokes equation is introduced. Concept of flow of energy between wave numbers and the effects of the inertia, pressure, and decay are discussed. One of the most important chapters is that dealing with the universal equilibrium theory. The logical arguments leading to the theory are clearly presented and the application to all types of turbulent flow are discussed. The physical consequences of statistical equilibrium and conditions for attaining it in practice are also given.

In the last two chapters, information derived from the experimental results is discussed. Titles of the chapters are: "Decay of the energy-containing eddies" and "The probability distribution of the velocity vector." The theoretical implications and the hypotheses which have been presented to explain these experimental results are presented in detail.

In reviewer's opinion this book is an excellent review of homogeneous turbulence and is recommended to all engineers working with fluid turbulence. Reviewer also sees it as an ideal textbook for advanced courses in fluid mechanics. A knowledge of tensor analysis is necessary for understanding the mathematical derivations, although none are presented in detail. Reviewer believes that inclusion of more experimental results would have made some sections clearer, but the book might have been too long had this been done.

W. D. Baines, Canada

2856. Wood, G. P., and Gooderum, P. B., Investigation with an interferometer of the flow around a circular-arc airfoil at Mach numbers between 0.6 and 0.9, NACA TN 2801, 80 pp., Oct. 1952.

Measurements were made of the state of the stream about an airfoil at free-stream Mach numbers resulting in mixed sub- and supersonic flow adjacent to the airfoil. Knowledge of this type of flow has been restricted to date by limitations of conventional instrumental and analytical techniques, and the independence of flow and measuring process by use of an interferometer is particularly advantageous.

This exploratory investigation of transonic flow about one airfoil provides the distribution of Mach number in the flow field, of pressure along the airfoil, of density ratio, pressure ratio, initial Mach number and wave angle along the shock wave. The authors note that their results do not confirm assumptions used by several previous authors, particularly relative to symmetry of the flow field, location of the point of free-stream pressure on the airfoil, and state of the stream before and after the shock wave near the airfoil. The authors do not evaluate the accuracy of their results in sufficient detail in the report to assure that the disagreements with earlier assumptions are all real.

E. W. Price, USA

2857. Wood, G. P., Experiments on transonic flow around wedges, NACA TN 2829, 34 pp., Nov. 1952.

Several aspects of transonic flow around the forward portions of wedge profiles were studied by means of interferometry. The

growth of the supersonic region at a sharp convex corner formed by two flat surfaces was observed. The pressure-drag coefficients of a wedge of 14.5° semi-angle were measured at Mach numbers of 0.768, 0.819, and 0.854 and were shown to be consistent with those of wedges of smaller angle, when plotted according to the transonic similarity law. Conditions at the bases of the shock waves that interacted with boundary layers on the wedge were measured. The method of characteristics was used to calculate the flow behind an experimentally determined sonic line, and the calculated flow field was compared with the measured flow field.

From author's summary by J. S. Rinehart, USA

2858. Lin, C. C., On the stability of the laminar mixing region between two parallel streams in a gas, NACA TN 2887, 50 pp., Jan. 1953.

This summary paper gives the principal results of an extensive mathematical investigation of the stability of a laminar mixing region to be published in *J. Math. Phys.* Oscillations occur only if the relative speed of the two streams is less than the sum of their velocities of sound, if at some point $d/dy(\rho dw/dy) = 0$, where y is a transverse coordinate, ρ the density, and w the dimensionless velocity, and if the velocity w_s at that point is subsonic relative to both streams. For the case of constant enthalpy with one stream at rest, the flow becomes completely stable at a free-stream Mach number of 1.7.

H. L. Dryden, USA

2859. Thwaites, B., Investigations into the effect of continuous suction on laminar boundary-layer flow under adverse pressure gradients, Aero. Res. Coun. Lond. Rep. Mem. 2514, 23 pp., 1946, published 1952.

Theoretical treatment is given of various problems of laminar boundary-layer flow with suction. The main problem is that of flow separation with constant suction and constant adverse pressure gradient. The method utilizes the momentum integral and leads to a solution only for small amounts of suction. Continuous suction over a region near the leading edge is shown to permit the attainment of high lift coefficients with small suction velocities. Author states that numerical data applied to various airfoils will be given in a later report.

A. M. Kuethe, USA

2860. Loftin, L. K., Jr., and Wilson, H. B., Jr., A rapid method for estimating the separation point of a compressible laminar boundary layer, NACA TN 2892, 19 pp., Feb. 1953.

Using Stewartson's transformations [*Proc. roy. Soc.*, 1949], authors derived a relation between the nondimensional velocity gradients $F_c = (L/U_{co}) (dU_c/dx_c)$ in the compressible, and $F_i = (L/U_{co}) (dU_i/dx_c)$ in the incompressible plane, viz., $F_c/F_i = 1 + \frac{1}{2}(\gamma - 1)M_o^2$. Here U = velocity, x = abscissa, L = equivalent flat-plate length $\approx \int_0^{x_c} (U_c/U_{co})^{0.75} dx_c$ (approximate transform of Doenhoff's approximate relation, title source, 671), M = Mach number and $\gamma = c_p/c_v$ = ratio of specific heats at constant pressure and volume. The subscripts i = incompressible, c = compressible, o = reference condition taken at point of maximum velocity.

Starting from Doenhoff's relation between F_i and $(U_{is} - U_{is})/U_{is}$ (subscript s = separation), the position of the laminar separation point was estimated for compressible flow, where classical boundary-layer assumptions are valid. From F_c , F_i and U_{is}/U_{is} are found. U_i/U_{is} is given as a function of M and U_c/U_{co} . So, U_{is} , and therefore x_{cs} , are known. Results are compared with those from Howarth's analysis and with Stewartson's application of Howarth's method. Considerable differences exist in the values of $(U_{co} - U_{cs})/U_{co}$ for all Mach numbers (due to introduction of numerous approximations). However, differences in

$(U_{is} - U_{is})/U_{is} = (U_{eo} - U_{eo})/U_{eo}$ are not too large, so that a rough estimation of U_{eo} seems possible if U_{is} is known. Results show that separation point moves forward if M increases.

H. Wijker, Holland

2861. Lin, C. C., On Taylor's hypothesis and the acceleration terms in the Navier-Stokes equations, *Quart. appl. Math.* 10, 4, 295-306, Jan. 1953.

An assumption that is used often in the analysis of turbulent velocity fluctuations observed at a fixed point is that the fluctuations are caused by a certain spatial distribution of velocity being translated past the point at the speed of the mean flow; with this assumption some of the spatial properties of the turbulence can be determined from the reading of a single hot-wire anemometer. The principal purpose of the paper is to examine explicitly the theoretical justification for this assumption in order to see, in particular, if it may be used in shear flow. In the case of isotropic turbulence, the author finds, with the aid of the hypothesis that the relation between fourth- and second-order velocity product mean values is as for a jointly normal distribution of the velocities at two points, that the criterion for the assumption to be accurate at large Reynolds numbers is that $5u^2/U \ll 1$. In shear flow, an immediate necessary (but not sufficient) condition for the assumption to be valid is that $|v(dU/dy)| \ll |U(\partial u/\partial x)|$ (with usual notation), but unfortunately this will not usually be satisfied; author shows that an equivalent criterion for an individual Fourier component may be satisfied if the wave length is small compared with the lateral scale of the flow.

G. K. Batchelor, England

2862. Uberoi, M. S., and Kovasznay, L. S. G., On mapping and measurement of random fields, *Quart. appl. Math.* 10, 4, 375-393, Jan. 1953.

Paper gives a theoretical foundation of the general relations between measurements and actual properties of turbulent flow. The consideration is applied to velocity measurements by hot-wire anemometer in isotropic turbulence, and density field measurements by shadow method in the wake of a body.

J. Rotta, Germany

2863. Liepmann, H. W., Aspects of the turbulence problem, *ZAMP* 3, 5, 6; 321-342, 407-426; Sept., Nov. 1952.

Survey article with excellent bibliography discusses stationary stochastic processes, turbulence effects upon linear systems, turbulent transport phenomena, and isotropic turbulence.

M. J. Goglia, USA

2864. Rott, N., Compressible laminar boundary layer on a heat-insulated body, *J. aero. Sci.* 20, 1, 67-68, Jan. 1953.

Authors show that the Illingworth-Stewartson transformation for reducing a two-dimensional heat-insulated laminar compressible boundary layer to a corresponding incompressible problem can be generalized to the case of arbitrary Prandtl number and viscosity-temperature relation, provided that two approximations regarding variations across the boundary layer are admissible. These approximations are: (1) A quadratic interpolation formula for the temperature distribution as a function of local velocity, similar to the exact solution for Prandtl number equal to unity; (2) a proportional increase of the viscosity coefficient with the local absolute temperature, using, however, the correct wall viscosity and temperature as the reference quantities. Some results of application to compressible boundary layer in the manner of a previous paper [Rott and Crabtree, *AMR* 5, Rev. 2902] are presented.

S. F. Shen, USA

2865. Lin, T. C., and Street, R. E., Effect of variable viscosity and thermal conductivity in high-speed slip flow between concentric cylinders, *NACA TN* 2895, 122 pp., Feb. 1953.

At pressures where mean free paths approach some characteristic dimension of objects, slip flow occurs. Zero velocity and equal wall and stream temperatures at any surface are then not observed. Problem can be treated using a method of Chapman and Cowling. Authors extend a solution of Schamberg's to include variation in viscosity and heat conduction with temperature. Friction factor and heat-transfer coefficient are found from original formulas using an effective mean free path and viscosity coefficient. Pressures are modified in a more complicated way. No experimental results are available at present to test either theory.

W. Griffith, USA

2866. Schwartzberg, M. A., and Braslow, A. L., Experimental study of the effects of finite surface disturbances and angle of attack on the laminar boundary layer of an NACA 64A010 airfoil with area suction, *NACA TN* 2796, 35 pp., Oct. 1952.

Description of an airfoil with a porous surface and internal suction. The effect of the suction on the stability of the boundary layer is studied with two- and three-dimensional perturbations, and found to be small. The size of the perturbation necessary to produce transition is not increased very much by area suction.

The drag coefficient, with suction, is slightly improved at low lift, but area suction is found unfavorable at high angles of attack.

R. Betchov, USA

2867. Berndt, S. B., Approximate calculation of the influence of wall boundary layers upon the blockage interference in a high-speed wind tunnel, *Flygtekn. Försöksanst. Medd.* no. 45, 12 pp., 1952.

The wall boundary layers in a subsonic wind tunnel work to reduce the choking effect of a model. A simple approach to a theoretical explanation is used. It involves neglecting viscosity, linearization of the equations of motion, and substitution of wall layers of constant velocity for the boundary layers. Solutions are given for plane and axisymmetric flow. Results of numerical application to the case of a parabolic arc profile in plane flow confirm experimental results and indicate strong boundary-layer effects to occur at Mach numbers close to 1.

From author's summary by H. J. Allen, USA

2868. Martin, J. J., Graphical solution for "Turbulent boundary layer in compressible fluids," *J. aero. Sci.* 20, 2, 147-148, Feb. 1953.

Note presents rapid method of estimating local and mean friction coefficients. The method is based on equations of Van Driest [AMR 4, Rev. 3647], and utilizes nomograph of pertinent parameters. Limits on parameters are as follows: Mach number, 0 to 10; wall-to-free-stream temperature ratio, 1 to 20; free-stream Reynolds number, 10^5 to 10^9 . Estimates of heat-transfer coefficient may be found from friction coefficients by means of simplified formula given in both note and referenced paper.

Reviewer believes that time saved in utilizing aforesaid method compensates for possible inaccuracies in reading graphs.

H. L. Bloom, USA

2869. Tarján, G., Contributions to the classical theory of jiggling, *Acta Techn. Hung., Budapest* 6, 1/2, 79-121, 1953.

Theoretical study of the motion of solid particles in water does not apply closely to hydraulic jiggling, but is of qualitative value in understanding and controlling the process.

The motion of solid particles in water pulsating up and down is studied mathematically. Finkey's integration of the differential

equation of motion for turbulent flow is an approximation which only holds for small particle accelerations. Author also derives corrected relations for water velocity between the particles, and thus for the distance small particles are lifted. Curves are given for the design of hydraulic jigs. Author concludes that in jiggling uniformly sized particles, high decelerations are disadvantageous; with unclassified or equi-settling particles, high decelerations are advantageous.

C. F. Bonilla, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 2734, 2826, 2885, 2898, 2905, 2941)

2870. Garner, H. C., foreword by Bryant, L. W., **Swept-wing loading. A critical comparison of four subsonic vortex sheet theories**, *Aero. Res. Coun. Lond. curr. Pap.* 102, 47 pp., 10 figs., 1952.

Author presents the results of calculations, by various methods, of the wing lift slopes and local aerodynamic centers of four families of ideally thin swept wings. The primary object is to compare the merits and accuracy of the Kuchemann, Weissinger, Multhopp, and vortex lattice theories. Calculated results are compared with local aerodynamic centers as given by the elliptic quarter-chord point and with the lift-curve as given by a semi-empirical relationship. Agreements are within 5% with the exception of very small taper ratios for the former, and with the exception of sweep angles greater than 60° or aspect ratios less than 3/2 of the latter. Time estimates are given for the various methods of calculation. Although no exact solutions are available, author presents both qualitative and quantitative arguments which indicate that the Multhopp theory for swept wings [*RAE Rep. Aero 2353*, Jan. 1950] provides the most reliable solutions for swept-wing loadings in potential subsonic flow. The work reported is a first step in a coordinated program for the theoretical and experimental study of the effects of thickness and boundary layer upon wing air-load distributions.

T. F. O'Brien, USA

2871. Kordes, E. E., and Houbolt, J. C., **Evaluation of gust response characteristics of some existing aircraft with wing bending flexibility included**, *NACA TN 2897*, 31 pp., Feb. 1953.

A two-degree-of-freedom analysis (vertical motion and wing bending in fundamental mode) is applied to four different airplanes for a variety of weight conditions, gust and flight parameters. Results are presented in the form of dynamic response factors for acceleration and bending moment. Substantial dynamic overstress is obtained in some cases. A rule of thumb is suggested for judging when airplanes may be considered rigid for discrete gust analysis. A limited correlation of some acceleration results with flight data is presented.

G. Isakson, USA

2872. Riley, D. R., **Effect of horizontal-tail span and vertical location on the aerodynamic characteristics of an unswept tail assembly in sideslip**, *NACA TN 2907*, 39 pp., Feb. 1953.

An investigation has been conducted in the Langley stability tunnel on a vertical-tail model with a stub fuselage in combination with various horizontal tails.

The results of the investigation indicate that the induced loading carried by the horizontal tail produced a rolling moment about the point of attachment to the vertical tail which was strongly influenced by horizontal-tail span and vertical location. The greatest effect of horizontal-tail span on the rolling-moment derivative of the complete tail assembly was obtained for horizontal-tail locations near the top of the vertical tail.

The addition of a horizontal tail to the fuselage-vertical-tail

combination produced the greatest increase in the magnitude of the lateral-force derivative when the horizontal tail was located near either tip of the vertical tail.

Span loadings which were reduced to the static-stability derivatives were calculated for each configuration tested by applying the well-known finite-step method used for wings to the intersecting surfaces of the vertical and horizontal tails. The stability derivatives obtained by this method were in good agreement with wind-tunnel results.

From author's summary by W. Z. Stepniewski, USA

2873. Fischer, A., **Model investigations for the determination of wind-pressure distributions on cooling towers** (in German), *Schweiz. Bauztg.* 71, 2, 15-18, Jan. 1953.

Measurements were made at Reynolds number 4×10^5 on a body of revolution consisting of two truncated cones joined by a cylindrical section. For simulating vertical ribs on the tower, 96 wires were placed on the model during some measurements. Results show that wind pressure is largely increased by roughness. Known pressure distribution on rough cylinders of infinite length is applicable except at the top and bottom of the tower, where considerable deviations exist. Author believes that more measurements are necessary for investigation of further details of flow.

F. Keune, Sweden

2874. Palme, H. O., **Aerodynamic problem in construction of supersonic airplanes** (in Swedish), *Tekn. Tidskr.* 82, 20, 469-475, May, 1952.

Paper covers the aerodynamics of jet planes with speeds up to Mach numbers of about 1.5, and describes some of the numerous problems in obtaining a satisfactory over-all performance of the airplane throughout this wide speed range. The main flight mechanical problems dealt with are (a) performance analysis, including landing speed, maximum horizontal speed, and ceiling; (b) stability analysis, including loose and locked stabilizing surfaces; and (c) structural strength analysis. Author discusses qualitatively in some detail drag forces, air-intake losses, and center-of-pressure movements. Finally, paper gives an approximate aerodynamic comparison between the sweptback wing type and the delta wing type. Paper is general and essentially qualitative in its description of the subject.

T. A. Mortensen, USA

2875. Stepniewski, W. Z., **Some aerodynamic problems in helicopter design**, *Aircr. Engng.* 24, 283, 266-270, Sept. 1952.

Aerodynamic design problems affecting helicopter performance are discussed. Consideration is given to hovering, high speed, and range for rotors driven by reciprocating and jet engines. Range performance is compared with that of fixed-wing aircraft, and means for improvement of helicopter range are reviewed.

A. H. Flax, USA

2876. Press, H., and Mazelsky, B., **A study of the application of power-spectral methods of generalized harmonic analysis to gust loads on airplanes**, *NACA TN 2853*, 48 pp., Jan. 1953.

General relations for linear systems between power spectrums of random input disturbance and output response are used to relate spectrum of airplane load in rough air to spectrum of atmospheric gust velocity. Power spectrum of loads is shown to provide a measure of load intensity in terms of standard deviation (root mean square) of load distribution for an airplane in flight through continuous rough air. For case of load output having normal distribution, which appears from experimental evidence to apply to homogeneous rough air, standard deviation is shown to describe probability distribution of loads or proportion of total time that load has given values.

Two selected series of calculations are presented. In the first, standard deviations of loads in continuous rough air described by an assumed power spectrum are calculated for systematic variations in frequency and damping characteristics of airplane response to a step-gust input. Results obtained indicate that loads in rough air are particularly sensitive to variations in damping characteristics of oscillatory response to a step gust and largely independent of variations in frequency. In second application, standard deviation of loads is calculated for selected variations of each of several airplane geometric and aerodynamic parameters of an idealized and stable transport-type airplane. Standard deviations obtained are compared with results derived by conventional techniques of using calculated peak response to an idealized and representative discrete gust. Results indicate that, for stable configurations, both methods of analysis yield results that are consistent to a first approximation.

From authors' summary by G. Isakson, USA

2877. Slaymaker, S. E., and Gray, R. B., Power-off flare-up tests of a model helicopter rotor in vertical autorotation, NACA TN 2870, 36 pp., Jan. 1953.

The report deals with experiments on the vertical descent of a helicopter model in autorotation and the power-off flare-up due to an increase in collective pitch. During the tests the time histories of the following quantities were measured: (a) Rate of descent and autorotative speed as function of disk loading; (b) vertical displacement, vertical rate of descent, and rotor speed in a flare-up initiated by an increase in the collective pitch.

For the latter part of the test the following parameters were varied: inertia number of blade ($\gamma = 2.8$ and 5.3), disk loading (0.2 to 0.9 lb/ft²), time required to change collective pitch from zero to final value (0.2 to 3.0 sec), and the final collective blade pitch (10.5 to 15 deg).

Test data indicate that, for a given disk loading and rotor inertia, an increase in rate of change of blade-pitch angle and in amount of final blade-pitch angle results in a more effective flare maneuver. The variation of rotor speed during the flare maneuver is essentially linear. As expected, an increase in rotor inertia and a decrease in disk loading increase the effectiveness of the maneuver.

In the appendix, a semiempirical method of predicting the flare-up characteristics has been presented. Sample calculations were made for two full-scale helicopters. In addition, the theory was checked with the experimental results. The theory agrees fairly well with the tests as far as the final minimum rate of descent is concerned. With regard to the actual time history of the flare-up, however, the agreement is not quite satisfactory, especially for the lower final blade-pitch angles and the larger inertia number of blade investigated. G. J. Sissingh, USA

2878. Lange, R. H., Langley full-scale-tunnel investigation of the maximum lift and stalling characteristics of a trapezoidal wing of aspect ratio 4 with circular-arc airfoil sections, NACA TN 2823, 24 pp., Nov. 1952.

Author presents low Mach number data on an unswept wing of aspect ratio 4 having 10% thick circular-arc airfoil sections. Lift, drag, and pitching moment are included at Reynolds numbers ranging from 3,270,000 to 7,670,000. He includes the effects of a full-span leading-edge flap and both full-span and partial-span trailing-edge flaps. Tuft studies are presented for the basic wing and for the wing with flaps deflected. The maximum lift of the basic wing was 0.63 and a value of 1.39 was obtained with the best combination of leading-edge and trailing-edge flaps. The wing was insensitive to Reynolds number for the range of Reynolds number investigated. R. M. Crane, USA

2879. Baxter, J. R., A generalized treatment of manoeuvrability, Aero. Res. Lab., Austral., Rep. A.76, 47 pp., Oct. 1952.

Equations for steady maneuver at constant forward speed in a vertical plane are developed. This type of maneuver can be approximately reproduced in flight during a pullout from a dive. "Stick fixed" and "stick free" maneuvers are analyzed, and the treatment, which makes use of Laplace transforms, is sufficiently general for application to both conventional and tailless aircraft. Effects of springs and weights in the elevator circuit, as well as effect of elevator deflection on lift, are taken into account.

M. Morduchow, USA

2880. Mirels, H., Lift of highly swept wings, J. aero. Sci. 20, 3, 210-211, Mar. 1953.

The corrected integral equation for the solution of lift of highly swept wing, first derived by A. Robinson [AMR 6, Rev. 598], is given. Some limiting solutions are discussed. S. I. Pai, USA

2881. Adams, G. J., and Dugan, D. W., Theoretical damping in roll and rolling moment due to differential wing incidence for slender cruciform wings and wing-body combinations, NACA Rep. 1088, 11 pp., 1952.

A method of analysis based on slender-wing theory is developed to investigate the characteristics in roll of slender cruciform wings and wing-body combinations. The method makes use of the conformal mapping processes of classical hydrodynamics which transform the region outside a circle and the region outside an arbitrary arrangement of line segments intersecting at the origin. The application of the method has shown: (1) That the damping in roll and the rolling moment due to differential incidence of both pairs of opposite surfaces of the cruciform wing-body combinations are practically independent of the body-diameter maximum-span ratio up to a value of this ratio of 0.3. (2) That the damping in roll of the cruciform wing-body arrangement is only 62% greater than that for a corresponding planar wing-body combination. (3) That the rolling moment, resulting from differential incidence of both pairs of the opposing surfaces of the cruciform wing-body arrangement, is only 52% greater than that for a corresponding planar wing-body combination. (4) That the rolling effectiveness (wing-tip helix angle per unit surface deflection) of the cruciform wing-body arrangement having four equally deflected panels is therefore 94% of the corresponding planar wing-body combination.

The present work extends AMR 4, Rev. 1736.

From authors' summary by Y. C. Fung, USA

2882. Williams, J., Some investigations on thin nose-suction aerofoils, parts I and II, Aero. Res. Coun. Lond. Rep. Mem. 2698, 28 pp., Apr. 1950, published 1952.

The stalling properties of some thin nose-suction airfoils already tested have been examined, and further theoretical investigations have been carried out on thin airfoils especially designed to give high lift with nose-slot suction.

From author's summary by P. Schwaar, Switzerland

2883. Miele, A., Optimal flight trajectories of jet-propelled airplanes (in Italian), Acrotecnica 32, 4, 206-219, Aug. 1952.

General equations [AMR 4, Rev. 3954] for velocity distribution V with height z are applied to jet-propelled airplanes if the minimum time, minimum fuel consumption, or the steepest climbing from one condition (V_1, z_1) to another (V_2, z_2) is required. Engine speed and weight of aircraft are assumed constant; thrust and fuel consumption of the engine are functions of height only; the airplane polar is a quadratic parabola; the curvature of the path and squares of climbing angle are neglected. The flight tra-

jectory is composed of an initial and a final section, depending on the conditions at both ends, while the flight velocity and Mach number distribution of the middle part are given by a bicubic equation. Optimum Mach number is determined by two fundamental aircraft parameters: the ratio x between minimum aircraft drag in level flight and engine thrust, and the velocity of level flight with best gliding angle. Optimum Mach number distribution and acceleration are given by graphs. The effect of various simplifications is considered in an appendix. A. Kuhelj, Yugoslavia

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 2871, 2901, 2903)

2884. Influence of tuned dampers on flexure-aileron flutter. Part I. Frazer, R. A., and Jones, W. P., Theoretical investigation on the influence of tuned damping devices on flexure-aileron flutter; Part II. Jones, W. P., Some further calculations on the influence of tuned damping devices on flexure-aileron flutter; Part III. Scruton, C., Dunsdon, D. V., and Ray, P. M., Experiments on the effect of tuned damping devices on flexure-aileron flutter, *Aero. Res. Coun. Lond. Rep. Mem.* 2559, 41 pp., Sept. 1946, published 1952.

In part I, a general theory has been developed for investigating the influence of damping devices of various types on flexure-aileron flutter. The numerical applications refer to a large transport aircraft, and they are restricted to the case of a mass-balanced aileron-carried damper. From the diagrams given at the end of the part it is inferred that this type of damper would be unsatisfactory as a flutter preventive.

Part II supplements part I and gives results for a partly balanced and for a completely balanced aileron-damper system. It is concluded that tuned dampers of these types would also prove unreliable. Part III describes an experimental investigation into the effect on flexure-aileron flutter of a tuned damping device attached to the aileron. The results confirm the theoretical conclusion that the use of an aileron-carried damper would not be a reliable flutter preventive.

From authors' summary by S. F. Shen, USA

2885. van de Vooren, A. I., and Yff, J., Aerodynamic coefficients of an oscillating aerofoil with a pressure-seal balanced aileron, *Nat. LuchtLab. Amsterdam Rep. F.115*, 17 pp., Sept. 1952.

A pressure-sealed balanced aileron of an oscillating wing gives rise to additional nonstationary aerodynamic coefficients. These additional flutter coefficients are due to (a) the hinge moment caused by the pressure difference across the seal, (b) the oscillating flow through the seal chambers due to aileron oscillation (represented by an oscillating source-sink system) which affects the pressure on the entire wing, and (c) the effect of the oscillating seal chamber pressure on the hinge moment. These additional incompressible flutter coefficients are calculated in equation form and are presented in the notation of the NLL and that of Küssner-Schwarz.

H. P. Liepmann, USA

2886. Smith, F., and Hicks, W. D. T., Design of a simple electronic flutter simulator, *J. roy. aero. Soc.* 57, 505, 29-38, Jan. 1953.

The paper describes a two-degree-of-freedom flutter simulator which was intended as a prototype for a larger machine (six degrees of freedom) which has now been built. Coupled mechanical oscillations are replaced by corresponding electrical oscillations. The electrical circuits are normally varied step-by-step to correspond to increments in air speed until an undamped oscillation

is obtained in the electrical "degrees of freedom" which corresponds to flutter of the mechanical system. The frequency of the electrical oscillation is the flutter frequency, and the amplitudes and phase of each degree of freedom can be observed on voltmeters. Variations of flutter speed with any particular structural or aerodynamic parameter can readily be investigated.

For given basic aerodynamic and structural data, both simulators generally give good agreement with calculated flutter speeds.

Reviewer believes the six-degree-of-freedom simulator to be a particularly useful aid to flutter research.

W. P. Jones, England

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 2906, 2909)

2887. Reichel, R. H., Jet propulsion (in German), *Motortech. Z.* 14, 1, 2; 17-22, 52-56; Jan., Feb. 1953.

Author gives a simple one-dimensional momentum theory of jet propulsion devices. A historical review of intermittently and continuously operating air-breathing jet propulsion devices is given. A discussion follows of various means of thrust increase for turbojets, such as injection of water or methanol at the various components of the engines, or as afterburning. A table of American and English post-World War II turbojets is given in which the main characteristics of these engines are compared. Next, ramjets are discussed briefly. A discussion of the various rocket motors follows.

Although atomic energy as propulsive power is mentioned, only the known liquid- and solid-propellant rockets are discussed in detail. It is shown that, under certain assumptions, the thrust of a rocket increases with altitude. Conditions in the combustion chamber and the nozzle are discussed as well as the various methods of fuel injection systems. The arrangements of solid propellants for short and long duration of operation are also given. A comparative table of known German, American, and English rockets is presented. Combinations of the various thrust-producing devices are discussed next. Finally, over-all efficiency, thrust, fuel consumption, and range of the various power plants are compared.

T. P. Torda, USA

2888. Hammack, J. B., The aerodynamic design of supersonic propellers from structural considerations, *NACA TN* 2851, 21 pp., Dec. 1952.

A method of designing propellers which considers the spanwise distribution of cross-sectional area required to attain constant centrifugal stress over the blade is presented. Application of the method shows a reduction in propeller efficiency and blade power-absorbing ability as Mach number increases from 1.2 to 2.4.

From author's summary by A. Petroff, USA

2889. Hubbard, H. H., and Lassiter, L. W., Sound from a two-blade propeller at supersonic tip speeds, *NACA Rep.* 1079, 9 pp., 1952.

Propeller noise is a current problem of the aircraft industry; greater propeller speeds suggest louder noises. Knowledge of these noises should be important for design of future airplanes and aircraft installations. Authors measured noises from a 2-blade, 47-in. diam propeller in tip Mach numbers from 0.75 to 1.30. Graphs are presented of frequency spectra, wave shapes, and over-all sound levels at angles of 60°, 90°, and 120° with axis of rotation. Most measurements are at 30 ft. Gutin's theory, satisfactory to predict sound pressures at subsonic tip speeds, is adequate at supersonic tip speeds only for the lower harmonics. At

supersonic tip speeds, contrary to Gutin's theory, the higher harmonics decrease in intensity with increasing speed, and the over-all level remains constant at about 155 db at 4 feet from center of rotation and 130 db at 30 ft away. Authors state, but do not demonstrate, their curves are useful to estimate over-all levels if power, tip Mach number, and distance are known. Reviewer feels human significance could be added by emphasizing that sound levels at 30 ft can produce permanent loss of hearing, and at 4 ft can heat certain types of clothing.

E. Ackerman, USA

2890. Dzung, L. S., Pressure pulsation at the intake of a supercharged internal-combustion engine, *Brown Boveri Rev.* 39, 8/9, 295-305, Aug./Sept. 1952.

The system investigated consists of (1) the supercharging blower, (2) the engine, and (3) the distributing manifold which connects the blower to the engine cylinders. A mathematical analysis is derived and theoretical results compared with test conducted at Brown Boveri Co. The results are summarized by the author as follows: "The piston movement of an internal-combustion engine causes pressure pulsations at the inlet manifold, depending closely upon the performance characteristics of the supercharging blower. At acoustic resonance, the pulsation amplitudes may be very large. These phenomena are investigated here by the method of small perturbations. The question of whether a steady-state vibration exists can be determined by the stability criterion method. The rather self-evident condition that the pressure-volume curve of the supercharging blower must have a negative slope needs some essential readjustments on account of other influences. Such investigations help to coordinate the supercharger and the internal-combustion engine in the most favorable manner."

S. A. Andes, USA

2891. Jarre, G., Free vortex axial compressors (in Italian), *Monogr. sci. Aero.* no. 12, 47 pp., Sept. 1952.

Axial-compressor design is treated in interesting unconventional form: In part I, concise rigorous derivation of aerodynamic equations and general properties for inviscid axisymmetrical flows; in part II, application to free vortex machinery. Known results are obtained and presented in new form. Clear discussion is given of first stage and multistage design, together with numerical examples. Interest of paper lies in originality and neatness of presentation, but it suffers from limitation to free vortex case. Paper affords a typical example of the difficulty of using, at best efficiency, excellent human material in countries where low level of technical information and exchange has resulted from circumstances.

L. Crocco, USA

2892. McCoy, A. W., and Hopper, A. V., Blade adjustment in axial-flow compressor stages, *J. aero. Sci.* 20, 1, 34-48, Jan. 1953.

Authors present simplified theoretical analysis showing effectiveness of stator and rotor-blade controls in providing volume regulation in an axial-flow compressor.

B. Smilg, USA

2893. McBride, E. J., Blade profiles, *Proc. First U. S. nat. Congr. appl. Mech.*, June 1951; J. W. Edwards, Ann Arbor, Mich., 699-704, 1952.

Paper is an extension of an earlier method by Weinig ["Die Stromung um die Schaufeln von Turbomaschinen," Barth, Leipzig, 1935]. Potential flow of an incompressible fluid through a cascade of profile is treated by studying flow through a cascade of equivalent flat-plate profiles possessing the desired velocity distribution. Both profiles are transformed to same unit circle mapping plane, which allows velocity components to be put into

"plotting function" whereby the velocity distribution may be described. A criterion is given to insure a closed profile with a sharp trailing edge.

Reviewer believes that other methods, which also account for compressibility, may be more suitable for actual computations, due to their greater state of development [e.g., Costello, *AMR* 3, Rev. 941; 4, Rev. 3633].

Reviewer protests increasing tendency of authors to omit essential derivations and to present only results of their investigations.

H. N. Abramson, USA

2894. Zinner, K., The acceleration behavior of a diesel motor with an exhaust-driven supercharger (in German), *Motortech. Z.* 12, 2, 3; 41-44, 64-66; Feb., Mar. 1952.

In many diesel-engine applications, where exhaust turbosupercharging is used to increase the power capacity, the performance characteristics are of special interest at sudden load variations. Author concentrates his investigation on three load cases: (1) Increasing torsional moment at nearly constant engine rpm, where only the turbosupercharger has to accelerate; (2) accelerating the engine with supercharger at constant torsional moment; (3) acceleration from running at low speed without load to full speed and load. The calculations and experiments show that an exhaust turbosupercharged engine can master sudden load variations as well as, and in some cases better than an unsupercharged engine with comparable power capacity. A low turbocompressor momentum of inertia is of greatest importance.

G. Huss, Sweden

2895. Clark, P. M., Mechanical pumps for high-temperature liquid metal, *Ann. Meet. ASME*, New York, Dec. 1952. Paper 52-A-94, 6 pp., 6 figs.

Operational experiences are presented for two centrifugal pumps handling liquid sodium and liquid-sodium potassium up to temperatures of 750 F. Some information on efficiency and total head obtained with these pumps in functions of fluid flow rate and numbers of revolution is also included.

E. R. G. Eckert, USA

2896. Escande, L., and Nougaro, J., Sudden closure of the discharge of the turbines at the head of a tailrace conduit flowing full without surge tank (in French), *C.R. Acad. Sci. Paris* 234, 18, 1737-1739, Apr. 1952.

—, Progressive closure of the discharge of the turbines at the upstream end of a tailrace conduit flowing full without surge tank (in French), *ibid.* 234, 20, 1949-1951, May 1952.

—, Influence of the friction loss on the overpressures arising in a tailrace conduit flowing full (in French), *ibid.* 236, 8, 775-778, Feb. 1953.

—, and de Reynal, M., Measurements of overpressures in tailrace conduits flowing full (in French), *ibid.* 236, 9, 891-893, Mar. 1953.

—, Experiences with the operation of tailrace conduits flowing full (in French), *ibid.* 236, 12, 1234-1236, Mar. 1953.

Formulas are presented for cavity volumes, time intervals, and overpressures, due to return flow after sudden or gradual closure, at the upstream end of a conduit with aeration shaft or suction valve or without any device for pressure reduction.

Authors conclude: Sudden closure, giving rise to cavitation, causes occurrence of unacceptable pressures. Aeration shaft or suction valve gives considerable reduction. Increase of duration of closure has negligible effect when suction valves are applied, good effect with aeration shaft, and considerably reduces overpressures in conduits without any pressure-reducing device. Reductive effect of friction loss appears to be small.

Figures obtained by the formulas are compared with results of graphical computation of numerical examples. Agreement is good except in the case of small area of aeration shaft and short durations of closure, the graphical method giving lower figures for maximum pressures than the formulas. This discrepancy in extreme cases is pointed out but not analyzed.

The formulas are given without derivation, which, however, can be reconstructed from basic equations. For graphical computation procedures one is referred to publications of Bergeron; see, e.g., his "From water hammer in hydraulics till lightning in electricity" (in French), Paris, 1950.

In the last two notes, results are given of model experiments with the discharge suddenly cut off. Good agreement between theory and experiment was found in the case without any device and that with suction valve. Conduit with aeration shaft showed deviation on the safe side, compared with theory, in the higher ranges of maximum pressures and volumes of wasted overflow. Of the cavitation phenomenon, detailed description is illustrated by photographs taken with high-speed camera.

H. J. Shoemaker, Holland

Flow and Flight Test Techniques

(See also Revs. 2698, 2862, 2886, 2924)

2897. Wegener, P. P., and Lobb, R. K., An experimental study of a hypersonic wind-tunnel diffuser, *J. aero. Sci.* 20, 2, 105-110, Feb. 1953.

Experimental tests of a two-dimensional, variable-area diffuser in the NOL 12-by-12-cm hypersonic wind tunnel over a Mach number range of 5.9 to 9.6 are reported and discussed. Each of the two adjustable diffuser walls is composed of three "linked" plates (the first of which is linked to the nozzle exit) positioned by three electrically operated jacks.

Pressure-distribution measurements, schlieren spark photographs, and over-all pressure ratios for starting and maintaining hypersonic flow for several diffuser configurations are presented. Results indicate that a sharp throat followed by a three-degree plane wall divergence gave optimum pressure recovery, 1.8 to 2.3 times that recovered by a Pitot tube placed in the tunnel test section. Air condensation in the test section appeared to have small influence on diffuser performance.

Reviewer believes the results can be most helpful to researchers designing or building hypersonic tunnel equipment.

R. E. Bolz, USA

2898. Ross, J. G., and Lock, R. C., Wind-tunnel measurements of yawing moment due to yawing (n_r) on a 1/5.5 scale model of the Meteor Mark F.III, *Aero. Res. Coun. Lond. Rep. Mem.* 2791, 31 pp., 1947, published 1952.

This useful 1947 paper presents a critical review and application of the damped-oscillation method for measuring n_r (damping due to yawing) in a wind tunnel, and a comparison with results from "snaking" motion in full-scale flight. Attention is paid to dry friction, boundary-layer conditions, local separations, rudder-trailing-edge conditions, fuselage-fin fairing (dorsals), and to nacelle-propeller effects. A comparison of the present development of the method with the NACA method using curved-channel flow is hampered by lack of detailed information on the latter.

M. V. Morkovin, USA

2899. Tucker, M., Combined effect of damping screens and stream convergence on turbulence, *NACA TN* 2878, 62 pp., Jan. 1953.

Wind-tunnel turbulence is often reduced by fine mesh screens

located in a low-speed settling chamber followed by a contracting passage to the test section. (The author uses the term contracting, even in supersonic flow, for convenience.)

A plane wave analysis of the effect of the screens and an axisymmetric contraction on the scale and intensity of the turbulence is made. The turbulent fluctuations are assumed to be incompressible and initially isotropic, and decay is neglected. Extensive computations are included.

An approximate calculation is given for the decay of axisymmetric turbulence and the results are used to correct the previous calculation. The results thus obtained agree qualitatively with the available experimental data. It is claimed that the theory is sufficiently accurate for engineering purposes.

W. Squire, USA

2900. Vos, A. S., Methods and instrumentation for temperature measurements and the use of N.T.C. resistance thermistors for this purpose (in Dutch), *Chem. Weekblad, Amsterdam* 49, 5, 68-76, 1953.

This review article is on a fairly elementary level. The subject matter is divided into general introduction, thermocouples, resistance thermometers, and semiconducting resistance thermometers such as thermistors. Comparison is made of the advantages and disadvantages of the various methods, and a brief summary of some of the more important articles is given. Of particular value to an American reader is the large number of references covering numerous European contributions. No original results are reported.

R. Smoluchowski, USA

2901. Melzer, R., Design of dynamically similar airplane models to be used for wind-tunnel flutter experiments (in French), *ONERA Publ.* no. 59, 52 pp., 1952.

Analysis of the structural principles to be followed in design and construction of airplane models used in vibration studies and flutter-testing techniques. Report deals broadly with the technical developments in this direction as carried out in Great Britain and Germany during World War II, with specific reference to the airplane models developed by Junkers Flugzeugbau in Dessau.

From author's summary by M. A. Dengler, USA

2902. Drew, D. A., The measurement of turbine stresses in aircraft engines in the laboratory, on the test bed and in flight, *Proc. Soc. exp. Stress Anal.* 10, 1, 187-202, 1952.

Paper describes equipment and measurement techniques used by Rolls Royce, Ltd., Derby, England. Stresses in the turbine buckets were determined by means of wire-wound strain gages utilizing Nichrome wire of 0.002-in. diam.

For the laboratory tests a magnetostrictive transducer was used to vibrate the turbine buckets. This equipment enabled adequately high stresses to be induced in the buckets up to frequencies of approximately 13,000 cps.

For rotating tests during test bed and flight investigations, low noise level, mercury-type slip rings were developed.

Results of stress measurements in turbine buckets were reviewed for a series of investigations of the Rolls Royce Derwent engine. It was established that vibratory energy was being fed from the buckets into the disks and through the disks to other buckets. Considerable reduction in bucket stresses was obtained by relative tuning of the buckets.

The test program established that previous failures of the Derwent V turbine buckets under service conditions were due to 48th-order stresses, which corresponded to the nozzle guide-vane order. An interesting but unexplained fact was that the nozzle guide-vane order stresses increased in flight, especially at high speed and low altitude, by a factor of about 3 to 1; whereas the

lower-order stresses were relatively unaffected by flight conditions
F. L. Wattendorf, USA

2903. Rost, J., Ground vibration tests on airplane models (in French), *ONERA NT* no. 13, 21 pp., 1952.

Report on experimental investigations carried out on an airplane model resembling the Dornier 217. Main objective of study is the determination of the structural and dynamical characteristics of the model in view of wing flutter, including mode shapes, natural frequencies, structural damping coefficients, and generalized masses. New methods of vibration excitation were employed.

From author's summary by M. A. Dengler, USA

2904. Annand, W. J. D., The resistance to air flow of wire gauzes, *J. roy. aero. Soc.* 57, 507, 141-146, Mar. 1953.

Paper presents an empirical method (based on correlated experimental data) of estimation of the pressure drop for gauzes with porosity from 0.27 to 1.00, over a range of Reynolds number from 20 to 600.

From author's summary by A. Petroff, USA

2905. Sadoff, M., On the use of a damped sine-wave elevator motion for computing the design maneuvering horizontal-tail load, *NACA TN* 2877, 32 pp., Jan. 1953.

Paper gives an alternative to the method of Pearson [AMR 1, Rev. 1024] and Kelley and Missall [AAF Air Tech. Service Command, Wright Field, *Tech. Rep.* 5185, 1945]. Operational methods are employed to find the tail-load response to an elevator input of the form $\Delta\delta(t) = A_1 e^{h_1 \omega_1 t} \sin \omega_1 t$. When the constants in the elevator function are adjusted to give variation of $\Delta\delta$ similar to that of Kelley and Missall, the tail loads are in good agreement. Author also investigates the effect of the frequency parameter ω_1 . With a given airplane load factor, an increase of ω_1 increases maximum values of $\Delta\delta$, $d\delta/dt$, and tail load. The computational procedure is simpler than that of Kelley and Missall, and the elevator function, being smooth, is more realistic.

Paper brings to six the number of different methods for computing horizontal tail loads. The previous papers are, in chronological order of publication: (1) Pearson [AMR 1, Rev. 1024]; Kelley and Missall, essentially the same, differing in the method of evaluating a Duhamel integral. (2) Pearson, McGowan, and Donegan [AMR 5, Rev. 845]. (3) Etkin and Woodward [AMR 4, Rev. 4563]. (4) Decker [AMR 5, Rev. 2459]. (5) Smith and Rea [AMR 6, Rev. 1996].

All these methods are based upon essentially the same differential equation, given by Pearson; but they differ in the method of solution. In papers 1, 5, and present one, an arbitrary variation of elevator angle is prescribed and adjusted to give the desired maximum load factor in the maneuver. The classical mathematical problem which results is then solved by standard methods, yielding time histories of airplane load factor, tail angle of attack, and tail load. In papers 2, 3, and 4, the inverse method is employed; i.e., the time history of airplane load factor is specified, and the variation in elevator angle and tail load computed from the basic differential equation. The latter method is inherently more direct and leads to a simpler mathematical problem, entailing far less computational labor. The results obtained are just as good.

B. Etkin, Canada

2906. Duncombe, E., A method of estimating optimum turbine operating conditions for a range of nozzle and blade angles, *Nat. aero. Establ. Canada*, R.17, 29 pp., 1952.

See AMR 4, Rev. 3356.

2907. Dreher, R. C., An airborne indicator for measuring vertical velocity of airplanes at wheel contact, *NACA TN* 2906, 19 pp., Feb. 1953.

One of the important factors governing landing-gear loads is the vertical velocity of the airplane. In general, vertical velocity is not accurately known, and NACA has undertaken a general study of obtaining accurate measurements of this quantity. Paper describes method developed for measuring vertical velocity by means of a trailing arm which is attached to landing gear and makes runway contact just prior to landing-gear wheel. Vertical velocity is determined from angular motion of trailing arm, using a small generator which translates motion into velocity measurements. Generator, which is mounted on vertical-velocity indicator, was calibrated at two positions of trailing arm.

Accuracy of vertical-velocity indicator was tested in Langley impact basin, where test specimen can be dropped at constant velocity while carriage is moving horizontally. Flight tests, involving 14 landings, were made using two-place single-engine trainer-type airplane of 5200-lb gross weight. With landing-gear inclination ranging from 9° to 19°, and with horizontal landing speed ranging from 95 to 125 fps, the vertical velocities recorded ranged from 0.5 to 3.2 fps at contact. An improved model which can be used with retractable landing gear was subsequently developed. Typical oscillographic records obtained during impact-basin and flight tests of the prototype, as well as sample records obtained with the improved design on a high-speed jet bomber, are included.

J. P. Michalos, USA

2908. Grossman, L. M., and Charwat, A. F., The measurement of turbulent velocity fluctuations by the method of electromagnetic induction, *Rev. sci. Instrum.* 23, 12, 741-747, Dec. 1952.

A method is described for measuring the component velocity fluctuations in the turbulent flow of a fluid by investigating the induced potential gradients in flow transverse to a steady magnetic field. Experimental equipment and procedures for calibration and reduction of data are discussed. Measurements of the rms component-velocity fluctuations in the radial and axial directions for the flow of water in a circular pipe are presented.

From authors' summary

Thermodynamics

(See also Revs. 2810, 2846, 2887, 2890, 2942)

2909. von Kármán, T., and Millán, G., The thermal theory of constant pressure deflagration, *Anniv. Vol. appl. Mech.*, dedicated to C. B. Biezeno: Haarlem, Antwerpen, Djakarta: N. V. De Technische Uitgeverij H. Stam, 1953, pp. 59-69. Hfl. 20.

Authors mathematically treat the problem of the determination of the normal velocity of combustion in a gas mixture. The problem is one-dimensional in space with temperature, density, chemical composition, and velocity as functions of the x coordinate only. The flow is laminar, with viscous forces neglected. The treatment considers only heat transfer and neglects diffusion. The fundamental equations are given and assumptions clearly stated. The resulting problem is an eigenvalue determination. The equations force authors to assume an ignition temperature if they are to realize a combustion wave with constant propagation velocity. The case of a first-order chemical reaction is then solved approximately. The conclusions are that the bulk of the chemical reaction occurs at temperatures relatively near the final temperature, practically independent of the choice of ignition temperature. The normal combustion velocity which makes the process stationary is practically constant and independent of the

choice of the ignition temperature, with the exception of ignition temperatures near the initial and final temperatures.

This paper has the usual clarity and insight of Dr. von Kármán. A similar approach, interesting for comparison purposes, was published by H. W. Emmons, Harvard University Computation Lab., Feb. 1950.

R. A. Gross, USA

2910. Kulakov, M. V., On the determination of thermic coefficients of insulating materials (in Russian), *Zh. tekhn. Fiz.* 22, 1, 67-72, Jan. 1952.

In order to determine the thermal properties of a given solid material, author advocates the use of nonstationary heat-conduction processes which he claims, from his experience, to be more precise and at the same time easier to handle than the corresponding stationary processes. He considers the following setup: The material to be tested is a plate of thickness $2R$, in contact at both faces ($x = \pm R$) with an infinitely extending standard material of known properties. At $x = 0, t = 0$, an amount of heat is then supplied instantaneously. The rise and subsequent fall of temperature at a given station within the standard body is observed, and the thermal coefficients of the tested sample deduced therefrom. The mathematical problem (solution of the parabolic partial-differential equation with proper regard to the initial condition and the boundary conditions where the two bodies meet) is solved by means of Laplace transformation along well-established lines.

A. von Baranoff, France

2911. Cullen, R. E., A nondimensional correlation of flame propagation at subatmospheric pressures, *Trans. ASME* 75, 1, 43-49, Jan. 1953.

Experimental investigation of Bunsen flame for two different mixtures, different burner sizes, and different pressures shows influence of burner size and pressure on flame speed. Flame speed for infinite burner size seems to depend logarithmically upon pressure. Influence of burner is tentatively explained by heat losses within the burner. Subsequent discussion by A. A. Putnam gives other possible explanation with the same consequences.

G. Guderley, USA

2912. van Itterbeek, A., Determination of the thermodynamic and kinetic properties of gases and condensed gases at low temperatures (in French), *Atti Conv. Internaz. Ultracust.* 1950, 218-228, 1951.

Measurements are reported of the velocity of sound at various pressures (about 0.5 atm) and temperatures (about 85 K) in mixtures of A-H₂, A-He, H₂-He, O₂-H₂, and H₂-He; also in some organic liquids. Measurements of the absorption of sound in gaseous hydrogen at low temperatures (60-300 K) are given. Calculations of second virial coefficients and adiabatic compressibilities are based on these measurements.

H. C. Brinkman, Indonesia

2913. ter Haar, D., The isotherms of an imperfect gas, *Proc. Camb. phil. Soc.* 49, part 1, 130-135, Jan. 1953.

Wergeland's liquid drop model of an imperfect gas is discussed by considering grand canonical ensembles.

P. Kriezis, Greece

2914. Codegone, C., On the thermal conductivity of gases and vapors (in Italian), *Monogr. Lab. Aero. Politecn. Torino* no. 300, 3 pp., 1952 = *Atti Accad. Sci. Torino* 86, 1951-1952.

The extension of the theorem of corresponding states to the thermal conductivity of gases and vapors produces results of the importance of various applications.

From author's summary by M. A. Dengler, USA

2915. Codegone, C., On the dynamic viscosity of gases and vapors (in Italian), *Monogr. Lab. Aero. Politecn. Torino* no. 293, 3 pp., 1952 = *Atti Accad. Sci. Torino* 86, 1951-1952.

Based on recent investigations on the viscosity of water vapor, considerations are extended to the general case of gases and vapors. From author's summary by M. A. Dengler, USA

2916. Messinger, B. L., Equilibrium temperature of an unheated icing surface as a function of air speed, *J. aero. Sci.* 20, 1, 29-42, Jan. 1953.

Author analyzes the factors which govern the equilibrium temperature of an insulated, unheated surface exposed to icing, and concludes that at least some ice can form on such a surface up to speeds of about 690 mph under normal low-temperature icing conditions. Previous work is extended by considering an important range of icing conditions of 32 F, wherein a new concept, the freezing fraction, is introduced. This concept is formulated as a basic variable which expresses the proportion of impinging liquid water freezing within the area of impingement. In addition, equations pertinent to the determination of equilibrium surface temperatures as a function of air speed are presented in convenient charts. These charts have been prepared primarily for an altitude of 20,000 feet, and cover appropriate ranges in values of liquid-water content and ambient-air temperatures.

N. R. Bergman, USA

2917. Yosida, K., On Brownian motion in a homogeneous Riemannian space, *Pacific J. Math.* 2, 263-270, 1952.

Let R be an n -dimensional, orientable, infinitely differentiable Riemannian space such that the group G of isometric transformations S^* of R onto R constitutes a Lie group transitive on R . Suppose, furthermore, that the subgroup of G leaving a point x of R invariant is compact. Consider a Markov process on R which is temporally and spatially homogeneous, that is, it has transition probabilities with the property $P(t, x, E) = P(t, S^*x, S^*E)$ for each $S^* \in G$. The author shows that the familiar continuity condition

$$t^{-1} \int_{d(x, y) > \epsilon} P(t, x, dy) \rightarrow 0$$

implies

$$\limsup t^{-1} \int_R \frac{d^2(x, y)}{1 + d^2(x, y)} P(t, x, dy) < \infty$$

as $t \rightarrow 0$ with $d(x, y)$ denoting the distance between x and y . From this the author concludes that for every $f(x)$ satisfying certain regularity conditions, one has

$$t^{-1} \left\{ \int_R f(y) P(t, x, dy) - f(x) \right\} \rightarrow a^i(x) f_i + b^{ij}(x) f_{ij}$$

subscripts denoting partial derivatives. This is, essentially, the backward diffusion equation. The point is that no artificial differentiability conditions on P are imposed.

Courtesy of Mathematical Reviews

W. Feller, Sweden

2918. Ruggeri, R. S., General correlation of temperature profiles downstream of a heated air jet directed at various angles to air stream, *NACA TN* 2855, 59 pp., Dec. 1952.

This is an extensive experimental study of the temperature distribution downstream of a heated air jet directed at angles 90°, 60°, 45°, and 30° to an air stream. It is found that at a given angle and jet diameter there are four types of temperature profile for different velocity ratios and mixing distances. Under all experimental conditions, the penetration coefficient (penetration

distance divided by $C^{1/2} D_j$, D_j being the orifice diameter and C the discharge-orifice flow coefficient) is proportional to

$$(\rho_j/\rho_0)^l (V_j/V_0)^m (s/C^{1/2} D_j)^n$$

Here ρ , V , s , denote, respectively, the density, velocity, and mixing distance, and j and o indicate, respectively, the conditions in the jet and of the free stream. By choosing the exponents l , m , n , good correlation of the experimental results has been obtained. Similar relations for the slope of the temperature profile are also found.

Y. H. Kuo, USA

2919. Saitô, N., and Sugita, M., Energy dissipation and entropy production in irreversible processes of dilute systems, *J. phys. Soc. Japan* **7**, 6, 554-559, Nov.-Dec. 1952.

Using differential equations for the distribution functions of molecules and the probability density, authors develop methods for calculating the entropy production in (1) the mixing of two ideal gases, (2) the settling under gravity of a dilute colloidal solution, (3) the application of an electric field across a dilute solution of polar molecules, and (4) problems involving viscosity in solutions of macromolecules.

From authors' summary by W. Hitschfeld, Canada

2920. Childs, M. E., Roose, R. W., Gilkey, H. T., and Konzo, S., Comparative performances of two warm-air perimeter systems and three convection systems, *Univ. Ill. Engng. Exp. Sta. Bull. Ser. no. 403*, 55 pp., Sept. 1952. \$0.60.

Authors report comparative results obtained in heating a low-cost basementless house, with a concrete slab floor laid on the ground, by means of the following types of warm-air heating systems: (1) A two-loop perimeter system; (2) a radial-feeder perimeter system (4 arrangements); (3) a conventional forced-air system with low wall registers; (4) a conventional forced-air system with high wall registers, and (5) a gravity system.

The results obtained with the three convection systems were not entirely satisfactory. The room-air temperature differentials from floor to ceiling were large for the high-wall and gravity systems; room-air temperatures at the 30-in. level were not uniform for the low-wall system; and floor-surface temperatures were low in the areas near the outside walls for all three systems.

The performance characteristics for the two-loop perimeter system were generally superior to those for the convection systems. Excellent control of room-air temperatures was provided by the use of a conventional room thermostat. However, some difficulty was experienced in balancing the system and in maintaining satisfactory floor-surface temperatures in the bedrooms.

Four arrangements of feeder ducts were studied in the investigation of the radial-feeder perimeter system—two arrangements with three feeders each, a four-feeder arrangement, and a five-feeder arrangement. The results obtained with the two arrangements in which feeder ducts were extended into the corners of the residence were much better than those obtained with the two-loop system; the room-air temperature differentials were markedly reduced, the cold areas on the floor surface were virtually eliminated, and no difficulty was experienced in balancing the system.

Y. S. Touloukian, USA

2921. Tsien, H. S., Physical mechanics, a new field in engineering science, *J. Amer. Rocket Soc.* **23**, 1, 14-16, 35, Jan.-Feb. 1953.

Author develops the thesis that the original aim of statistical mechanics was to deduce the properties of atoms and molecules from the observed properties of matter in bulk. Now, the engineer needs to know the properties of matter under extreme conditions, such as temperatures of 4000 K, where direct measurement

is difficult or impossible. At this temperature the average kinetic energy of the atoms is less than 0.52 electron volt, and the properties of atoms and molecules of this energy are well known. The function of statistical mechanics is thus reversed. "In many cases, the problem will not be solved by any means other than the most efficient and powerful method." This is illustrated by examples from the theory of imperfect gases.

F. R. N. Nabarro, England

Heat and Mass Transfer

(See also Revs. 2805, 2809, 2833, 2852, 2914, 2915)

2922. Dmitriev, A. A., A two-dimensional temperature problem in metal cutting (in Russian), *Zh. tekhn. Fiz.* **21**, 7, 832-841, July 1951.

Author replaces chisel by rectangular $0 < x < \infty$, $0 < y < l$; and piece of metal to be machined by quarterplane $l > x > -\infty$, $0 > y > -\infty$; the two fields touching along the segment $y = 0$, $0 < x < l$. At this segment, the temperatures must be equal, and it is assumed that a given quantity of heat is introduced by the cutting process. For both domains, the equation of heat conduction is solved with the aid of Fourier integrals (ordinary Laplace equation for the chisel; equation extended with a term depending on the movement of the material to be machined with a constant velocity v upward for the other domain), assuming a boundary heat loss proportional to the surface temperature. Different values of the coefficient are taken for the two domains. The dimensionless parameters determining the character of the solution are indicated. A rough comparison is made with some experimental data.

J. M. Burgers, Holland

2923. Vacca, Maria Teresa, Heat conduction of a thin annular plate bounded by two concentric circles (in Italian), *Atti Semin. Mat. Fis. Univ. Modena* **5**, 190-210, 1950-1951.

Paper deals with the general problem of distribution of temperature U in a thin annular plate bounded by two concentric circles of radii $r_1 < r_2$, $U(r, \vartheta, t)$ being a function of radius r , angle ϑ , and time t . Author starts from the well-known heat equation $\Delta U - (1/k)(\partial U / \partial t) = (a/k)(U - U_0)$ under initial and boundary conditions

$$\lim_{t \rightarrow 0} U(r, \vartheta, t) = f(r, \vartheta), U(r_1, \vartheta, t) = f_1(\vartheta, t), U(r_2, \vartheta, t) = f_2(\vartheta, t)$$

where k is the constant diffusivity, a another constant dependent on the exterior conductivity, and U_0 is the constant temperature of the medium surrounding the plate. Conditions indicated determine completely the function $U(r, \vartheta, t)$, which resolves also the problem of temperature distribution in a whole circular cylinder of infinite length under the assumption that the initial temperature of the cylinder and the temperatures on its internal and external surfaces are given. From this general solution, it is possible to get the solution of two special problems concerning radial heat flow in a circular pipe.

Z. Horak, Czechoslovakia

2924. Seban, R. A., and Drake, R. M., Local heat-transfer coefficients on the surface of an elliptical cylinder in a high-speed air stream, *Trans. ASME* **75**, 2, 235-239, Feb. 1953.

Authors report local heat-transfer coefficients, recovery factors, and pressure coefficients for zero and 5° angle of attack, at Reynolds numbers from 607,000 to 1,330,000 and upstream Mach numbers of 0.20 to 0.53. Paper advances the data available on boundary-layer heat transfer into the region of compressible flow and frictional heating. The cylinder had a 1:4 ratio of major to minor axis, the major axis being 5.5 in. long. It extended com-

pletely across a 7-in. square wind tunnel. Cylinder was made of bakelite, with 3 1-in. nichrome ribbons inlaid to form a heating surface with constant heat-rate characteristics. Local surface temperatures and pressures were measured. Working fluid was air at room temperature and pressure. Heat-transfer data are presented as Nusselt number divided by the square root of the Reynolds number versus a dimensionless peripheral distance from the forward stagnation point. This method yields a reasonably good correlation, which is further checked against a theoretical prediction with somewhat less success. C. L. Coldren, USA

2925. Cadiergues, R., Physical and geometrical principles of heat transfer by radiation (in French), *Chal. Industrie* 33, 34, 329, 330, 331; 395-410, 17-28, 43-54; Dec. 1952, Jan., Feb. 1953.

The material covered in this paper is of considerable value to those interested in radiation and illumination. A general survey of the theory of interchange of heat by radiation is neatly presented. Application of the theory to practical engineering problems is especially emphasized. Author covers the general theory of exchange functions and interchange factors and applications of the theory to interchange of heat by radiation between rectangular planes, parallel or perpendicular. Exchange functions are also tabulated for planes with various dimensions at various distances apart. Curves are given to obtain interchange factors between those planes. An analytical treatment is presented for the interchange of heat in an inclosure, and special consideration is given to solution of inclosures with two and three gray surfaces.

The last section of this analysis is devoted to engineering applications of radiant heating and comparison is given with convection heating systems. Paper contains emissivity data for metallic and nonmetallic materials. S. Eskinazi, USA

2926. Thomas, P. H., Absorption and scattering of radiation by water sprays of large drops, *Brit. J. appl. Phys.* 3, 12, 385-393, Dec. 1952.

Paper presents results of theoretical investigation of heat radiation through a layer of water spray of large drops. Multiple refractive scattering and absorption are considered in developing theory.

Numerical results are given for the transmission of thermal radiation. This is governed by the simple exponential extinction law if drops are large. When the product (absorption index) (drop radius) falls below about 3, absorption by the drop is incomplete and spray transmits more than is predicted by the exponential law. W. A. Wolfe, Canada

2927. Tessin, W., Heat transfer in bayonet-type tubes, *J. Amer. Soc. nav. Engrs.* 64, 4, 707-718, Nov. 1952.

The heat transfer between a fluid circulating within a tube assembly and a fluid passing over the outside of the outer tube is investigated. The tube assembly, a bayonet-type tube, consists of an inner and outer tube so that the circulating fluid passes through a circular passage, and then through an annular passage, or vice versa. Heat-transfer equations are presented for the temperature distribution for the cases where the circulating fluid within the assembly is hotter or colder than the system's environment, and where the fluid enters the inner or outer passages. The analysis shows that maximum or minimum temperatures of the fluid may occur within the system and that most heat is transferred if the inner tube is a thermal insulator. F. B. Schneider, USA

2928. Glass, W. J. S., Steam air heaters for marine water-tube boilers, *Trans. Inst. mar. Engrs.* 65, 1, 1-4, Jan. 1953.

The troubles of tubular gas air heaters in marine use are re-

viewed. These are corrosion, fouling, choking, explosions, and control. The performance of steam air heaters for 28,000-ton tankers is discussed.

One arrangement described is for one third of the feedwater from the low-pressure heater to be sent to the economizer. The other two thirds go from the low-pressure heater through the high-pressure heater to the high-temperature section of the economizer. The steam for the air heater then comes from the low-pressure bleed point.

The other arrangement described uses steam for air heating from both bleed points. The heaters are described. A. O. Flinner, USA

2929. Lovass-Nagy, Mathematical investigation of the process of cooling (in German), *Maschinenb.-Tech.* 2, 1, 7-11, Jan. 1953.

Graphs and formulas concerning thermal conduction in cylindrical bodies are compiled. Use of paper is diminished by minor but numerous errors. R. Eisenschitz, England

Acoustics

(See also Revs. 2700, 2797, 2889)

2930. Dingle, R. B., The velocity of second sound in various media, *Proc. phys. Soc. Lond. (A)* 65, part 12, 396A, 1044-1050, Dec. 1952.

Formulas are quoted for second sound velocity C_2 (velocity of temperature waves) known to exist to date only in liquid helium II. The formulas depend on "the effective mass-density of the excitations which contribute to the entropy" which, in turn, depends on the energy spectrum $E(p)$ (energy as a function of particle momentum) of the material, and are applied to calculate C_2 in non-ideal gases, insulators, conductors, superconductors, and ferromagnets. In each case, a formal assumption is made about $E(p)$, based on some model of the material and its elementary excitations; e.g., Debye waves in insulators, two-fluid model of superconductors, spin waves or collective electron model in ferromagnets, leading to formulas for $C_2(T)$ and numerical values. Six theoretical conditions on the properties of the medium and its excitations are noted which must be satisfied if second sound is to propagate. P. Marcus, USA

2931. Lilley, G. M., Westley, R., Yates, A. H., and Busing, J. R., On some aspects of the noise propagation from supersonic aircraft, *Coll. aero. Cranfield Rep.* no. 71, 37 pp., 30 figs., Feb. 1953.

The "sonic booms" which are observed after an aircraft has dived at supersonic speeds are shown to depend primarily on the shock-wave pattern formed by the aircraft. In the case of accelerated or retarded flight, these shock formations are shown to differ from the corresponding ones observed in steady flight. Some of these transient phenomena were studied with the aid of a hydraulic analogy and are related to the intense short-duration "booms" observed on the ground. It was concluded tentatively that the observed noise levels could be limited to 120 decibels for supersonic flight Mach numbers up to 3.0, provided such flights were restricted to altitudes greater than 22,000 feet and to distances of at least 4 miles from the nearest aircraft. H. H. Hubbard, USA

2932. Lacam, A., and Noury, J., Ultrasonic velocities in argon subjected to pressures up to 950 atm (in French), *C. R. Acad. Sci. Paris* 236, 4, 362-364, Jan. 1953.

The sound velocity at 900 ke in argon subjected to pressures

between 100 and 950 atm has been measured by the light-diffraction method. The velocity was found to increase steadily with rising pressure. The accuracy is claimed to be better than 1%; the temperatures were 24 and 25 C. F. E. Borgnis, USA

2933. Hall, R. G. N., and Sayce, L. A., On the production of diffraction gratings. II. The generation of helical rulings and the preparation of plane gratings therefrom, *Proc. roy. Soc. Lond. (A)* **215**, 1123, 536-550, Dec. 1952.

Paper describes the development at the National Physical Laboratory of Sir Thomas Merton's method of ruling helical diffraction gratings and of transferring them to plane surfaces. It is shown that the periodic errors inevitable in lathe-cut screw threads can be completely eliminated from helices up to 15,000 threads per inch, and that, by applying the replica process already described, each helix can yield an unlimited number of ghost-free plane gratings. It is also shown that, by ruling a grating as one continuous helical groove, it is easy to control the distribution of the diffracted energy and to avoid the casual errors which are commonly produced by reciprocating ruling engines.

From authors' summary

2934. Morris, W. E., Stambaugh, R. B., and Gehman, S. D., Ultrasonic method of tire inspection, *Rev. sci. Instrum.* **23**, 12, 729-734, Dec. 1952.

Author discusses principles of a method for detecting internal defects in pneumatic tires, such as ply separations, by the attenuation of transmitted ultrasonic waves. Equipment for carrying out tire inspections by this method is described. It includes provisions for inspecting a large range of tire sizes, scanning the tread region from shoulder to shoulder, and is suitable for operation by inspection personnel. A useful form for recording the results of a tire inspection is described. One of the principal applications for this method of inspection is the examination of tires to determine their suitability for recapping.

From authors' summary

2935. Nomura, Y., and Kawai, N., On the acoustic field by a vibrating source arbitrarily distributed on a plane circular plate, *Sci. Rep. Tôhoku Univ. (I)* **33**, 197-207, 1949.

Author starts from the "wave equation" for the velocity potential with cylindrical coordinates ρ , φ , z , and uses a solution which is a double infinite series of φ . The Fourier coefficients include infinite integral expressions involving Bessel functions of the first kind and exponential functions in the integrands. The series coefficients are determined by the velocity distribution on the circular plate. The aforesaid infinite integral expressions are evaluated using the addition theorem for Bessel functions. The acoustic field at a large distance from the circular plate is then calculated using Sonine-Gegenbauer formulas and asymptotic expressions for the Hankel functions. Finally, the pressure on the circular plate is calculated by the evaluation of the velocity potential on its surface. A formula for the radiation impedance is derived. Numerical values are given in several tables for the coefficients up to $k2\pi a/\lambda = 4$, where a is the radius of the plate and λ the wave length of sound in air.

Courtesy of Mathematical Reviews M. J. O. Strutt, Switzerland

2936. Nomura, Y., and Kawai, N., On the radiation of sound wave from a vibrating plane circular plate with a fixed circular baffle, *Sci. Rep. Tôhoku Univ. (I)* **33**, 208-215, 1949.

Paper pertains to the evaluation of the acoustic field caused by a circular plate vibrating axially in a fixed circular baffle. The velocity potential is expressed as an infinite series, the terms of which are infinite integrals involving products of Bessel functions

of the first kind in their integrands. The coefficients are determined from the given distribution of velocity over the surfaces of the plate and the baffle. Expressions are obtained for the field at a large distance from the plate and this is evaluated numerically in two sets of curves and in a polar diagram. From the pressure on the plate's surface, the radiation impedance is calculated and shown numerically in two sets of curves. The numerical values used in the diagrams are given in a table.

Courtesy of Mathematical Reviews

M. J. O. Strutt, Switzerland

2937. Eckart, G., Analysis of the echoes of sound waves produced by the stratification of the atmosphere (in French), *Acustica*, **2**, 6, 256-262, 1952.

The general theory of Eckart and Liénard [title source, **2**, 4, 157-161, 1952] is extended. Oblique incidence of a sound wave on a stratified medium is studied by the WKB method, and normal incidence by the perturbation method of Schelkunoff.

Reviewer notes with pleasure that the so-called WKB method, originally due to Lord Rayleigh [*Proc. roy. Soc. Lond. (A)* **86**, A 586, 207-226, 1912] is still capable of solving the problem for which it was developed.

F. R. N. Nabarro, England

2938. Smith, P. W., Jr., Systematic errors in indirect measurements of the velocity of sound, *J. acoust. Soc. Amer.* **24**, 6, 687-695, Nov. 1952.

An analysis is made of certain methods of measuring the velocity of sound. Specifically, attention is centered on those methods based upon measurements of the electrical driving-point impedance of the transducer and the observation of some phenomenon of that impedance which is recurrent as the position of the reflector in the acoustic chamber is changed. Consideration is given to the influences of small losses in the propagation of sound in the chamber and of the existence of nonplane wave fronts. It is shown that such measurements can be subject to significant systematic errors arising from the method itself. The nature and magnitude of such errors are discussed.

From author's summary by E. G. Fischer, USA

2939. Eckart, C., The theory of noise in continuous media, *J. acoust. Soc. Amer.* **25**, 2, 195-199, Mar. 1953.

Space-correlation functions are defined as the average overtime of the product $p(\mathbf{x}_1, t)p(\mathbf{x}_2, t - \tau)$, where p is the acoustic pressure at point in space \mathbf{x} and at time t . These functions are used to describe a restricted class of pressure fields which includes simple harmonic waves and noise. Author derives differential equations satisfied by the correlation functions. Intensity-frequency spectra for isotropic and nonisotropic pressure distributions are developed. Two problems are solved. The relationship between noise and turbulence is discussed. Reviewer believes that correlation functions, fruitful in radar, may be of value in acoustic problems, but distributions and problems treated in this article can be handled more easily by an infinite sum of simple harmonic functions. Reviewer feels that attempts to connect theory and turbulence may be misleading.

E. Ackerman, USA

Ballistics, Detonics (Explosions)

2940. Burgers, J. M., Some remarks on detonation and deflagration problems in gases, *Anniv. Vol. appl. Mech.*, dedicated to C. B. Biezeno; Haarlem, Antwerpen, Djakarta, N. V. De Technische Uitgeverij H. Stam, 1953, pp. 71-80. Hfl. 20.

This paper is an extension of discussion of uniqueness of the solution of equations for a weak deflagration wave given in

Courant and Friedrichs (p. 235). Author develops approximate solution for variation-of-state quantities through reaction zone of deflagration wave under assumptions that the chemical reaction is bimolecular, that number of moles of gas and mean specific heat at constant volume per unit mass at end of reaction are the same as at beginning, and that all gases are ideal. The velocity of the deflagration wave is determined by properties of the reacting gases.

R. E. Duff, USA

2941. Bryson, A. E., Jr., Note on the dynamic stability of a missile in rolling flight, *J. aero. Sci.* 20, 1, 69-71, Jan. 1953.

Author points out several errors in Ray E. Bolz's paper on the subject [title source, 19, 6, 1952]. He adds a corrected proof for the theorem: A missile that is dynamically stable for zero roll rate cannot be made unstable by the presence of a roll rate except under rather unusual conditions. The proof is similar to that of Bolz, but unessential forces and moments have been neglected and a simpler reference system and dimensionless quantities have been used.

O. Bottema, Holland

2942. Loeuille, E., Experimental determination of the ratio of specific heats of gases resulting from the burning of gunpowders (in French), *C. R. Acad. Sci. Paris* 236, 4, 358-360, Jan. 26, 1953.

When the "force" of gunpowder and its head of combustion at constant volume are known from experiment, the ratio of specific heats of the gaseous products of explosion can be computed if the chemical composition of the powder is known. The calculation depends on a simple energy balance, the explosion in a bomb being assumed to take place adiabatically and the gases being taken as ideal. For eight different military powders, the ratio of specific heats computed in this way agreed closely with values calculated directly from the known specific heats of the various gaseous products.

The "force" of the powder is equal to pV , where p is the pressure developed on explosion in a bomb of volume V .

R. L. Pigford, USA

2943. Bernd, R., Design and application of solid-fuel rockets (in German), *ZVDI* 95, 1, 12-16, Jan. 1953.

A nonmathematical review of German wartime developments. Some discussion is given of the difficulties found with very large grains.

H. H. M. Pike, England

2944. Diplock, B. R., Lofts, D. L., and Grimston, R. A., Liquid propellant rocket motors, *J. roy. aero. Soc.* 57, 505, 19-28, Jan. 1953.

Paper deals with the concepts related to the theory, design, and development of rocket motors. The following topics are analyzed: Performance of the rocket; design of the transformation nozzle; propellants; design of the combustion chamber and of the propellant feed system; development work before the test-bed stage is reached and work connected with the actual running of a rocket motor, with considerations on materials, valves, seals, rig-testing of components, arrangement of the test bed, instrumentation, and protection of the personnel.

A. Miele, USA

Soil Mechanics, Seepage

2945. Habib, P., The shearing strength of soils (in French), *Ann. Inst. tech. Bat. Trav. publics* 6, 61, 1-39, Jan. 1953.

Author describes compression, tension, and torsion tests on sands, silts, and clays to estimate effect of intermediate principal stress on strength. Results indicate greatest effect in sands where

drained angle of friction in compression of 32° compares with 25° measured in tension, while in torsion ϕ may reach 36° .

Effect of varying height: Diameter ratio of clay compression specimens confirms that ratio must lie between 1 and about 3.

To examine validity of effective stress hypothesis and current strength theories, author replaces pore water by benzene in a clay. Surprising result emerges that measured undrained angle of 20° and consolidated undrained angle of 36° compared with zero and 15° with water. Reviewer believes these results are due to incomplete saturation with benzene, but need for further research is indicated.

R. E. Gibson, England

2946. Kohler, K., On the difficulties of measuring friction coefficients and cohesion parameters of soils (in German), *Bautechnik* 30, 2, 41-42, Feb. 1953.

Author states that cause of the discrepancy between theoretical and experimental results in stability problems of earth masses lies in the character of shear tests. During a direct shear test, a slip surface of smallest resistance, composed of broken planes, will be developed at first; the resistance depends on the form of the shear box. As to the value of shearing resistance to be used in stability computations, author suggests the limit of proportionality of the stress-strain diagram, for the ultimate value applies to cases of geometrically similar circumstances in the test and reality.

Á. Kézdi, Hungary

2947. Németh, E., Model studies of water seepage, *Acta Techn. Hung.*, Budapest 3, 1/2, 131-176, 1952.

Article deals with experimental determination of the uplift force of seepage and of the velocity observable at its outlet.

Two model tests were carried out; the object of the first was the most economical arrangement of a larger dam relative to the number, location, and depth of sheet piles used. The resulting saving amounted to 30 times the costs of the model test. Results are in fair agreement with those obtained by calculation by Khosla's approximate method. The difference amounted to 2-4%.

The second test investigated the effect of sheet piles on the surface of drawdown produced by wells lowering the water table in foundation construction. The results led to the formulation of some general principles, but the tests have still to be continued.

From author's summary by T. Mogami, Japan

2948. Fröhlich, O. K., On the danger of sliding of the upstream embankment of an earth dam, caused by complete or partial discharge of the reservoir, *Trans. Fourth Congr. inter. Comm. Large Dams*, New Delhi, Jan. 1951, in 4 vols., vol. I, pp. 329-342. \$48 per set.

In discharging some French reservoirs, slides have occurred in the upstream slopes of the earth reservoir dams. Author calculates the change in the factor of safety of the embankments during discharging. The cross section of the sliding plane is presumed to be a circular arc. The slides have actually occurred in a stadium, where the author finds a factor of safety of about 1. It does not make much difference in the results whether or not the stress distribution along the sliding plane, as published earlier by the author [AMR 4, Rev. 483], is taken into account.

F. C. de Nie, Holland

2949. Hansen, J. B., A general plasticity theory for clay, *Géotechnique, Lond.* 3, 4, 154-164, Dec. 1952.

Author's theory assumes saturated clay with cohesion and internal friction, validity of Coulomb-Hvorslev failure criterion, linear increase of undrained strength with depth and sinusoidal variation of this strength with inclination of failure plane. The

theory is applied to determine "active" and "passive" pressures against smooth vertical wall and bearing capacity of infinite strip load. The result confirms correctness of Rankine and Prandtl solutions only when coefficient of earth pressure at rest is unity and strength constant with depth. With these restrictions, $\phi = 0$ procedure is shown to be valid provided shear strength is taken as half compression strength, but the rupture lines in Prandtl's problem, due to internal friction, are somewhat different from the ordinary figure. R. E. Gibson, England

2950. Le Nobel, J. C., Calculations of wing gate sewers (in Dutch), *Ingenieur* 64, 50, B.179-B.183, Dec. 1952.

Author's summary states: Special attention should be given to the dimensions of wing-gate sewers, controlling and quickly adapting the water level in the wing case for any functioning condition, considering the leak between wing and lock building, without their being too big. A specific example illustrates a theoretical method developed in the paper for calculating these dimensions.

Reviewer's opinion: Paper contains a detailed description of hydraulic operation of wing gates. Calculations give minimum dimensions. Uncertainty of hydraulic coefficients necessitates critical attitude toward calculated dimensions.

H. J. Schoemaker, Holland

2951. Hansen, V. E., Complicated well problems solved by the membrane analogy, *Trans. Amer. geophys. Un.* 33, 6, 912-916, Dec. 1952.

Methods of dealing with the flow of water to wells have tended to become more approximate with the increased complexity of the boundary conditions of specific aquifers. In reversing this trend, the author presents the membrane theory, describes experimental equipment and the results obtained from one of a series of experiments. Membranes are properly shaped to correspond to physical boundaries and deflected at the necessary points to correspond to operating wells. Resulting shape of membrane corresponds to shape of piezometric surface. Paper is particularly valuable to a ground-water user of "relaxation methods" for same class of problem, since it will assist in obtaining first approximation for the relaxation solution. Reviewer notes that paper would be more useful if it included a discussion of methods of properly tensioning membrane to simulate impermeable boundary, or boundary that changes from permeable to impermeable, etc.

R. G. Kazmann, USA

Micromeritics

(See Rev. 2869)

Geophysics, Meteorology, Oceanography

(See also Revs. 2916, 2937)

2952. Malkus, Joanne S., and Stern, M. E., The flow of a stable atmosphere over a heated island, Part I, *J. Meteor.* 10, 1, 30-41, Feb. 1953.

The island is considered as a two-dimensional strip represented by a finite Fourier series in such a way that it is repeated every 10 widths, but strips do not interfere. Uniform air stream stably stratified flows across, and the heating produces a superimposed circulation which is treated by perturbation methods. The result is unrealistic, but when a rigid lid is placed at a finite height the model corresponds in its dynamical properties with a probable case, and a reasonable result is obtained. In particular, a train of lee waves is induced and the wave crests are identified,

somewhat questionably in reviewer's opinion, with preferred sites for occurrence of cumulus. Observations made over Nantucket Island are called in as evidence for the qualitative conclusions.

The assumption that, though the heating is effected by convection, the transport of momentum by the same elements can be ignored, is acceptable at this stage. Authors point out that the form of heat input used is not easily made realistic because of discontinuities at upwind coastline. An associated difficulty arises because the air stream is stably stratified until it reaches the island where the layers heated possess neutral stability and then perturbation methods are inappropriate. Qualitatively, lee waves are readily seen to be generated, but quantitative predictions are impossible at present with a realistic model. As is well known, the assumption of a steady state of indefinite periodic motion creates somewhat unreal difficulties through attempts to simplify the mathematics in the sea-breeze problem. Authors promise a part II in which some of these troubles are tackled.

R. S. Scorer, England

2953. Lina, L. J., and Ricker, H. H., Jr., Measurements of temperature variations in the atmosphere near the tropopause with reference to airspeed calibration by the temperature method, *NACA TN* 2807, 23 pp., Oct. 1952.

An air-speed calibration system devised by J. A. Zalovick [NACA TN 2046, 1950] required, for the method to be accurate, that the air temperature must remain constant and the rate of change of temperature with pressure be smaller than the slope representing the adiabatic relationship. These criteria were evaluated in a limited number of aircraft flights made during 1950 near Langley Field, Va., over a Mach range from 0.6 to 0.8 in the vicinity of the tropopause. A special low-lag, high-recovery thermometer and static- and impact-pressure units were used to record air temperature and pressure. Very briefly, the results showed that the combination of occasional observed horizontal (constant pressure) temperature variations of up to $\frac{3}{4}$ °C in $\frac{1}{2}$ mile and the generally noninsignificant temperature variations with height (pressure) encountered in atmosphere having mean temperature profiles between the NACA standard atmosphere and the isothermal condition, were not favorable to an accurate air-speed calibration. While the reviewer has had no experience with such air-speed-calibration systems, he feels that the temperature observations are of interest and certainly corroborate previous observation of high-level atmospheric inhomogeneities in temperature and moisture. [Crain and Gerhardt, *Proc. IRE*, 41, 2, 284-290, Feb. 1953]. J. R. Gerhardt, USA

2954. Newell, H. R., Jr., and Siry, J. W., Rocket upper air research, *J. Amer. Rocket Soc.* 23, 1, 7-13, Jan.-Feb. 1953.

Author describes methods for measuring properties of the upper atmosphere with the help of rockets. Results are given of measurements of pressure, density of air temperature, O_3 -concentration, density of electrons, solar spectra, and decrease of magnetic field as a function of altitude up to 200 km. Conditions of wind and dissociation in high altitudes are discussed. Paper includes 34 references. S. F. Erdmann, Sweden

2955. Knighting, E., On the equation of diffusion in the atmosphere, *Quart. J. Mech. appl. Math.* 5, part 4, 423-431, Dec. 1952.

Author presents an account of the formal solutions of the equation of turbulent diffusion in two dimensions with the boundary conditions encountered in meteorological problems, and indicates how particular problems may be attacked. Special attention is given to the possible values of the exponent when the diffusion

coefficient is proportional to a power of the height above a given datum level. The applicability of the solutions can only be determined after accurate quantitative studies in the lower atmosphere.

H. Lake, USA

2956. Benioff, H., and Gutenberg, B., The response of strain and pendulum seismographs to surface waves, *Bull. seism. Soc. Amer.* 42, 3, 229-237, July 1952.

The azimuthal and phase response characteristics of strain seismographs, as compared with those of pendulum instruments, are outlined briefly. The combined response of two mutually perpendicular horizontal strain seismographs is noted as being independent of azimuth for Rayleigh waves, and zero for Love waves. The use of various combinations of strain instruments and of horizontal and vertical pendulum instruments in the identification of surface waves and in the determination of their azimuths is described. Several examples of seismograms are presented as illustrations.

F. G. Blake, Jr., USA

2957. Regula, W., Investigations on the foundations of a topographical method of weather prediction (in German), *Meteor. Rdsch.* 5, 9/10, 164-168, Sept./Oct. 1952.

In the construction of a predicted topography of constant-pressure levels, an advective tendency map is very useful in locating the areas of greatest change in the relative topography during the subsequent 24 hours. In determining the 24-hour areas of change by means of the curvatures of the relative topography, further synoptic data are required because without these the correlation is poor. Since the variation of the absolute topography is the sum of the variation in ground pressure and the variation of the relative topography, the correlation between them is of particular significance and has been computed by the author for thousands of pairs of values. From this correlation, valuable deductions are made on the size of the variations in particular areas. The predicted topography proved to be nearly as accurate as the ground prognostic chart.

In searching for a tendency map (3-hr variations) of the absolute topography, the nonadiabatic variations also were investigated, and insurmountable difficulties were encountered in taking into account the heat-transfer coefficients. However, the sums of the adiabatic and nonadiabatic variations could be determined. An analysis was made of the weather situation for December 28 and 29, 1951, and yielded insight into the causes of a hurricane low-pressure area.

W. C. Johnson, Jr., USA

2958. Eliassen, A., The quasi-static equations of motion with pressure as independent variable, *Geofys. Publ. Norske Vid.-Akad. Oslo* 17, no. 3, 44 pp., 1949.

In meteorological work, especially under the added assumption that the vertical pressure distribution is hydrostatic, it is often convenient to use pressure as a vertical coordinate. The author systematically transforms all the usual meteorological equations to such a system, and discusses their use and approximate solution.

C. Truesdell, USA

2959. Quan, B., and Wenham, H. G., Some tests of a refrigerated rotating cylinder for measuring ice accretion, *Nat. aero. Establ. Canada LR-45*, 9 pp., May 1952.

In measuring the liquid water content of icing clouds, the problem of runoff theoretically imposes an upper limit to the range of values that can be measured by the conventional rotating-cylinder technique. Present tests employed an internally refrigerated rotating cylinder to measure liquid-water-content values. Results indicated that water contents well above the

theoretical limit can be measured by such a technique. Comparative data for a conventional rotating cylinder, however, are not presented.

N. R. Bergman, USA

2960. Surinov, Yu. A., Radiative transfer in an absorbing and scattering atmosphere (in Russian), *Izv. Akad. Nauk SSSR Old. tekhn. Nauk* no. 9, 1331-1352, Sept. 1952.

This is the first part of a paper dealing with mathematical methods in the phenomenological theory of radiative transfer; application to engineering problems is envisaged. Paper starts with the fundamental concepts required for specifying a field of radiating energy. Considering the presence of gray bodies in an atmosphere propagating radiation, the continuity equation for the energy is obtained in the form of an integrodifferential equation. Application of this equation to characteristic special problems is discussed.

R. Eisenschitz, England

2961. Mineur, H., Investigations on the possible influence of the moon on atmospheric precipitations (in French), *Ann. Géophys.* 8, 3, 294-308, 1952.

Some apparently quite sound statistical analysis indicates a significant relation between rainfall and lunar phase for two French seacoast locations. For two inshore locations no such relation was evident. It is shown in the paper, however, that ocean tides cannot be responsible for this phenomenon, nor is any alternative explanation set forth.

Joanne S. Malkus, USA

2962. Munk, W., and Groves, G., The effect of winds and ocean currents on the annual variation in latitude, *J. Meteor.* 9, 6, 385-396, Dec. 1952.

Astronomical observations during the past 50 years have shown an annual motion of the axis of instantaneous rotation of the earth so that the north pole describes an elliptical excursion of 20-ft mean diameter. Part of this deviation is due to seasonal variations in the distribution of matter, mainly of air masses. Snow and changes in the vegetation also play some role. The estimated effects of wind [cf. Munk, W. H., and Miller, R. L., *Tellus* 2, p. 93, 1950] indicate that it is mainly the pressure against the Himalayas during the monsoon periods that can produce deviations of the order of magnitude observed.

H. Merbt, Sweden

2963. Arnason, G., A baroclinic model of the atmosphere applicable to the problem of numerical forecasting in three dimensions. I, *Tellus* 4, 4, 356-373, Nov. 1952.

Assuming the atmosphere is polytropic, a three-dimensional baroclinic model of the atmosphere is presented. This model is completely determined by four parameters, three of which specify the field of temperature covering both troposphere and stratosphere. Allowing the variation of temperature lapse rate in the horizontal, the integrated vorticity equation along the vertical, which is no longer simply appreciated, is written down. After examination of the coefficients of the said vorticity equation in relation to the parameters of the model, it is concluded that, in the middle latitudes, the level of mean wind may vary from 2.5 to 6.5 km and the level of nondivergence is slightly higher than the former level, the difference amounting to 0.5 ~ 3.0 km. On the average, the level of mean wind lies below and the level of nondivergence above the 500-mb level, which might be used to approximate each of the two levels mentioned.

The reviewer believes that the method used for determining the level of mean wind [Eq. (4.4)] as well as the level of nondivergence [Eq. (4.12)] cannot be accepted as the authorized one.

H. Arakawa, Japan

Lubrication; Bearings; Wear

2964. Heidebroek, E., Behavior of viscous liquids with specific reference to lubricating liquids in narrow gaps [Das Verhalten von zähen Flüssigkeiten, insbesondere Schmierflüssigkeiten, in engen Spalten], Berlin, Akademie Verlag, 1952, 39 pp. DM 5.80.

For many years author's laboratory has carried out extensive research on lubricants, bearing design, and allied equipment. This pamphlet summarizes results pertaining particularly to physical properties of lubricants, describes test equipment devised to determine such properties, and lists precautions to be taken in experiments. Conventional hydrodynamic theory fails when, under certain operating conditions, gap between bearing and shaft becomes so small that thickness of oil film approaches molecular dimensions; a better theory based on better knowledge of structure of viscous liquids is urgently needed. Author emphasizes that such knowledge at present is far inferior to that for gases and crystalline solids, and inadequate for engineering purposes; he suggests further research along chemical-physical (or even atomic physics) lines. Some helpful discussion on general bearing and lubrication theory is included.

C. W. Smith, USA

2965. Charnes, A., and Saibel, E., On the solution of the Reynolds equation for slider-bearing lubrication. II, *Trans. ASME* 75, 2, 269-271, Feb. 1953.

By a suitable transformation of Reynolds equation in two dimensions, the pressure distribution for constant viscosity is shown to be related in a simple way to the pressure distribution when the viscosity is a definite function of pressure. The true oil-film pressure for isothermal conditions may thus be determined from known constant viscosity solutions. The load capacity of the film can then be found by integration. This process requires numerical integration, but a lower limit to the load capacity is given. The actual pressure in the film is always greater than that obtained, assuming constant viscosity.

The results for a square slider are obtained in a numerical example which shows an increase in the maximum pressure of from 5050 psi for constant viscosity to 9450 psi, and an increase in load capacity of from 9650 lb to more than 12,000 lb for the same film dimensions, due to an increase in viscosity with pressure.

W. O. Richmond, Canada

2966. Heidebroek, E., Substitution of ball or roller bearings for journal bearings [Richtlinien für den Austausch von Wälzlagern gegen Gleitlager], Dresden, Dresdener Verlagsgesellschaft, 1950, 95 pp. DM 4.20.

The various factors to be considered when a substitution of bearings is contemplated are listed. Comparison is made of capabilities of ball, roller, and journal bearings, including some unconventional arrangements. Endurance, structural features, lubrication, and bearing calculations are discussed. A convenient table facilitates reference to the large number of useful charts, diagrams, and sketches. With help of this book, the machine designer will not overlook any important factors concerned with substitution of bearings, and will find a large number of different possibilities presented.

C. W. Smith, USA

2967. Charnes, A., Saibel, E., and Ying, S. C., On the solution of the Reynolds equation for slider-bearing lubrication. V. The sector-thrust bearing, *Ann. Meet. ASME*, New York, Dec. 1952. Paper 52-A-34, 17 pp., 2 figs.

Solution is obtained of the Reynolds equation for sectorial slider in which film thickness varies exponentially in tangential and radial directions. This is an extension of solution for rectangular slider obtained in AMR 5, Rev. 1594, where it is shown that results for exponentially varying film thickness agree very well with results for linear thickness variation. Solution is obtained in the form of a rapidly converging series. Constant viscosity is assumed. Values are obtained for pressure distribution, center of pressure, frictional moment, and oil flow.

This exact solution should prove valuable in the design of plane sectorial thrust bearings.

W. O. Richmond, Canada

Marine Engineering Problems

2968. Some unusual ship and machinery defects, their investigation and cure. A symposium of short papers, *Trans. Inst. mar. Engrs.* 65, 2, 13-37, Feb. 1953.

Papers treat detection and types of defects in boiler air heater and economizer, superheater tubes, steam drum, condenser, turbofeed pump, turbogenerator, diesel generator, steering gear, stern gland packing and shafting and hulls (due to vibrations). It is evident that when searching for an unusual cause of failure, the best way is to start with meticulous attention to detail.

Papers also describe unusual repairs to damaged main engine crankshafts, main engine crankpin, main steam turbine, and rudder pintle bearing. Repairs to a large crankshaft and to a large crankpin were carried out without removing them from the engine, thus saving about £50,000 and about three months' earnings, as compared with the orthodox method of repair.

E. Steneroth, Sweden

2969. Chambliss, D. B., and Boyd, G. M., Jr., The planing characteristics of two V-shaped prismatic surfaces having angles of dead rise of 20° and 40°, *NACA TN* 2876, 38 pp., Jan. 1953.

An investigation was conducted to determine the principal planing characteristics of two V-shaped surfaces having angles of dead rise of 20° and 40°. The data indicate that, for a given condition of load, speed, and trim, the wetted length, distance of center of pressure from trailing edge, and drag increase with an increase in the angle of dead rise.

From authors' summary by M. S. Macovsky, USA

2970. Edstrand, H., Model tests on optimum diameter for propellers, *Medd. SkeppsProvAnst. Göteborg* no. 22, 32 pp., 1953.

Four propeller sizes were tested on models (about 1:20 scale) of a fast cargo ship, a Liberty ship, and a tanker. Adapted to the same ship-loading conditions, and in conformance with Troost's B 4.40 series, these propellers had constant disk-area and blade-thickness ratios. For the fast cargo ship a 5% reduction from the open-water optimum diameter reduced the shaft power about 1%, while for the slower ships a 7% reduction decreased the power about 4%. Consideration of cavitation susceptibility indicates the changes are smaller for constant developed blade area.

J. M. Robertson, USA